



该文档主要有两个部分组成：第一部分涉及对外经贸大学GVC数据库（UIBE GVC数据库）中全球价值链链长R语言代码的解析；第二部分则是基于对外经贸大学GVC数据库提供的算法求解出本文计算双边价值链链长的计算思路与代码。

需要说明的是，UIBE GVC数据库当前只提供了一国部门对世界的全球价值链生产链长的计算代码（基于R语言）。本文以此为基础，将相关代码拆解至双边国家层面，从而计算出本文的双边价值链链长。在计算出相关的国家-部门-目的地层面的相关数据后，本文还加总至国家-部门-世界层面进行验证，所得结果与UIBE GVC数据提供的国家-部门-世界的结果一致。因此，本文的代码解析与计算方式具备可行性。

拆解思路参考：祝嘉良、方颖：《全球贸易格局动态下多边贸易合作与中国制造业“稳链”——基于对中国制造业供应链生产风险来源的测算》，《数量经济技术经济研究》2024年第11期。

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一：对UIBE的VA_GVC(即PLv_GVC分母)代码解析—— $V(B-L)Y$ 分解至国家_行业.世界

UIBE的代码：

```
z1 <- diag( VC )%*%(B%*%B-L%*%L)%*%diag(rowSums( Y ))  
z2 <- diag( VC )%*%(B-L)%*%diag(rowSums( Y ))  
PLv_GVC<- rowSums(z1)/rowSums(z2)
```

VA_GVC分解至国家_行业.世界——V(B-L)Y分解

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix}$$

$$\hat{V} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix} = \begin{pmatrix} \sum_{u=s}^t Y_1^{su} \\ \sum_{u=s}^t Y_2^{su} \\ \sum_{u=s}^t Y_1^{ru} \\ \sum_{u=s}^t Y_2^{ru} \\ \sum_{u=s}^t Y_1^{tu} \\ \sum_{u=s}^t Y_2^{tu} \end{pmatrix}$$

$$B - L = \begin{pmatrix} B_{11}^{ss} - L_{11}^{ss} & B_{12}^{ss} - L_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} - L_{21}^{ss} & B_{22}^{ss} - L_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} - L_{11}^{rr} & B_{12}^{rr} - L_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} - L_{21}^{rr} & B_{22}^{rr} - L_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} - L_{11}^{tt} & B_{12}^{tt} - L_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} - L_{21}^{tt} & B_{22}^{tt} - L_{22}^{tt} \end{pmatrix}$$

VA_GVC分解至国家_行业.世界——V(B-L)Y分解

$$(B-L)\widehat{\text{rowSums}(Y)} = \begin{pmatrix} B_{11}^{ss} - L_{11}^{ss} & B_{12}^{ss} - L_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} - L_{21}^{ss} & B_{22}^{ss} - L_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} - L_{11}^{rr} & B_{12}^{rr} - L_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} - L_{21}^{rr} & B_{22}^{rr} - L_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} - L_{11}^{tt} & B_{12}^{tt} - L_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} - L_{21}^{tt} & B_{22}^{tt} - L_{22}^{tt} \end{pmatrix} \begin{pmatrix} Y_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_2^t \end{pmatrix}$$

$$(B-L)\widehat{\text{rowSums}(Y)} = \begin{pmatrix} (B_{11}^{ss} - L_{11}^{ss})Y_1^s & (B_{12}^{ss} - L_{12}^{ss})Y_2^s & B_{11}^{sr}Y_1^r & B_{12}^{sr}Y_2^r & B_{11}^{st}Y_1^t & B_{12}^{st}Y_2^t \\ (B_{21}^{ss} - L_{21}^{ss})Y_1^s & (B_{22}^{ss} - L_{22}^{ss})Y_2^s & B_{21}^{sr}Y_1^r & B_{22}^{sr}Y_2^r & B_{21}^{st}Y_1^t & B_{22}^{st}Y_2^t \\ B_{11}^{rs}Y_1^s & B_{12}^{rs}Y_2^s & (B_{11}^{rr} - L_{11}^{rr})Y_1^r & (B_{12}^{rr} - L_{12}^{rr})Y_2^r & B_{11}^{rt}Y_1^t & B_{12}^{rt}Y_2^t \\ B_{21}^{rs}Y_1^s & B_{22}^{rs}Y_2^s & (B_{21}^{rr} - L_{21}^{rr})Y_1^r & (B_{22}^{rr} - L_{22}^{rr})Y_2^r & B_{21}^{rt}Y_1^t & B_{22}^{rt}Y_2^t \\ B_{11}^{ts}Y_1^s & B_{12}^{ts}Y_2^s & B_{11}^{tr}Y_1^r & B_{12}^{tr}Y_2^r & (B_{11}^{tt} - L_{11}^{tt})Y_1^t & (B_{12}^{tt} - L_{12}^{tt})Y_2^t \\ B_{21}^{ts}Y_1^s & B_{22}^{ts}Y_2^s & B_{21}^{tr}Y_1^r & B_{22}^{tr}Y_2^r & (B_{21}^{tt} - L_{21}^{tt})Y_1^t & (B_{22}^{tt} - L_{22}^{tt})Y_2^t \end{pmatrix}$$

$$\widehat{V}(B-L)\widehat{\text{rowSums}(Y)} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \begin{pmatrix} (B_{11}^{ss} - L_{11}^{ss})Y_1^s & (B_{12}^{ss} - L_{12}^{ss})Y_2^s & B_{11}^{sr}Y_1^r & B_{12}^{sr}Y_2^r & B_{11}^{st}Y_1^t & B_{12}^{st}Y_2^t \\ (B_{21}^{ss} - L_{21}^{ss})Y_1^s & (B_{22}^{ss} - L_{22}^{ss})Y_2^s & B_{21}^{sr}Y_1^r & B_{22}^{sr}Y_2^r & B_{21}^{st}Y_1^t & B_{22}^{st}Y_2^t \\ B_{11}^{rs}Y_1^s & B_{12}^{rs}Y_2^s & (B_{11}^{rr} - L_{11}^{rr})Y_1^r & (B_{12}^{rr} - L_{12}^{rr})Y_2^r & B_{11}^{rt}Y_1^t & B_{12}^{rt}Y_2^t \\ B_{21}^{rs}Y_1^s & B_{22}^{rs}Y_2^s & (B_{21}^{rr} - L_{21}^{rr})Y_1^r & (B_{22}^{rr} - L_{22}^{rr})Y_2^r & B_{21}^{rt}Y_1^t & B_{22}^{rt}Y_2^t \\ B_{11}^{ts}Y_1^s & B_{12}^{ts}Y_2^s & B_{11}^{tr}Y_1^r & B_{12}^{tr}Y_2^r & (B_{11}^{tt} - L_{11}^{tt})Y_1^t & (B_{12}^{tt} - L_{12}^{tt})Y_2^t \\ B_{21}^{ts}Y_1^s & B_{22}^{ts}Y_2^s & B_{21}^{tr}Y_1^r & B_{22}^{tr}Y_2^r & (B_{21}^{tt} - L_{21}^{tt})Y_1^t & (B_{22}^{tt} - L_{22}^{tt})Y_2^t \end{pmatrix}$$

VA_GVC分解至国家_行业.世界——V(B-L)Y分解

$$\widehat{V}(B-L)\text{rowSums}(Y) = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \begin{pmatrix} (B_{11}^{ss} - L_{11}^{ss})Y_1^s & (B_{12}^{ss} - L_{12}^{ss})Y_2^s & B_{11}^{sr}Y_1^r & B_{12}^{sr}Y_2^r & B_{11}^{st}Y_1^t & B_{12}^{st}Y_2^t \\ (B_{21}^{ss} - L_{21}^{ss})Y_1^s & (B_{22}^{ss} - L_{22}^{ss})Y_2^s & B_{21}^{sr}Y_1^r & B_{22}^{sr}Y_2^r & B_{21}^{st}Y_1^t & B_{22}^{st}Y_2^t \\ B_{11}^{rs}Y_1^s & B_{12}^{rs}Y_2^s & (B_{11}^{rr} - L_{11}^{rr})Y_1^r & (B_{12}^{rr} - L_{12}^{rr})Y_2^r & B_{11}^{rt}Y_1^t & B_{12}^{rt}Y_2^t \\ B_{21}^{rs}Y_1^s & B_{22}^{rs}Y_2^s & (B_{21}^{rr} - L_{21}^{rr})Y_1^r & (B_{22}^{rr} - L_{22}^{rr})Y_2^r & B_{21}^{rt}Y_1^t & B_{22}^{rt}Y_2^t \\ B_{11}^{ts}Y_1^s & B_{12}^{ts}Y_2^s & B_{11}^{tr}Y_1^r & B_{12}^{tr}Y_2^r & (B_{11}^{tt} - L_{11}^{tt})Y_1^t & (B_{12}^{tt} - L_{12}^{tt})Y_2^t \\ B_{21}^{ts}Y_1^s & B_{22}^{ts}Y_2^s & B_{21}^{tr}Y_1^r & B_{22}^{tr}Y_2^r & (B_{21}^{tt} - L_{21}^{tt})Y_1^t & (B_{22}^{tt} - L_{22}^{tt})Y_2^t \end{pmatrix}$$

$$\widehat{V}(B-L)\text{rowSums}(Y) = \begin{pmatrix} V_1^s(B_{11}^{ss} - L_{11}^{ss})Y_1^s & V_1^s(B_{12}^{ss} - L_{12}^{ss})Y_2^s & V_1^s B_{11}^{sr}Y_1^r & V_1^s B_{12}^{sr}Y_2^r & V_1^s B_{11}^{st}Y_1^t & V_1^s B_{12}^{st}Y_2^t \\ V_2^s(B_{21}^{ss} - L_{21}^{ss})Y_1^s & V_2^s(B_{22}^{ss} - L_{22}^{ss})Y_2^s & V_2^s B_{21}^{sr}Y_1^r & V_2^s B_{22}^{sr}Y_2^r & V_2^s B_{21}^{st}Y_1^t & V_2^s B_{22}^{st}Y_2^t \\ V_1^r B_{11}^{rs}Y_1^s & V_1^r B_{12}^{rs}Y_2^s & V_1^r (B_{11}^{rr} - L_{11}^{rr})Y_1^r & V_1^r (B_{12}^{rr} - L_{12}^{rr})Y_2^r & V_1^r B_{11}^{rt}Y_1^t & V_1^r B_{12}^{rt}Y_2^t \\ V_2^r B_{21}^{rs}Y_1^s & V_2^r B_{22}^{rs}Y_2^s & V_2^r (B_{21}^{rr} - L_{21}^{rr})Y_1^r & V_2^r (B_{22}^{rr} - L_{22}^{rr})Y_2^r & V_2^r B_{21}^{rt}Y_1^t & V_2^r B_{22}^{rt}Y_2^t \\ V_1^t B_{11}^{ts}Y_1^s & V_1^t B_{12}^{ts}Y_2^s & V_1^t B_{11}^{tr}Y_1^r & V_1^t B_{12}^{tr}Y_2^r & V_1^t (B_{11}^{tt} - L_{11}^{tt})Y_1^t & V_1^t (B_{12}^{tt} - L_{12}^{tt})Y_2^t \\ V_2^t B_{21}^{ts}Y_1^s & V_2^t B_{22}^{ts}Y_2^s & V_2^t B_{21}^{tr}Y_1^r & V_2^t B_{22}^{tr}Y_2^r & V_2^t (B_{21}^{tt} - L_{21}^{tt})Y_1^t & V_2^t (B_{22}^{tt} - L_{22}^{tt})Y_2^t \end{pmatrix}$$

VA_GVC分解至国家_行业.世界——V(B-L)Y分解

$$\text{rowSums}(\widehat{\hat{V}}(B-L)\widehat{\text{rowSums}(Y)}) = \begin{pmatrix} V_1^s B_{11}^{ss} Y_1^s + V_1^s B_{12}^{ss} Y_2^s + V_1^s B_{11}^{sr} Y_1^r + V_1^s B_{12}^{sr} Y_2^r + V_1^s B_{11}^{st} Y_1^t + V_1^s B_{12}^{st} Y_2^t - V_1^s L_{11}^{ss} Y_1^s - V_1^s L_{12}^{ss} Y_2^s \\ V_2^s B_{21}^{ss} Y_1^s + V_2^s B_{22}^{ss} Y_2^s + V_2^s B_{21}^{sr} Y_1^r + V_2^s B_{22}^{sr} Y_2^r + V_2^s B_{21}^{st} Y_1^t + V_2^s B_{22}^{st} Y_2^t - V_1^s L_{21}^{ss} Y_1^s - V_1^s L_{22}^{ss} Y_2^s \\ V_1^r B_{11}^{rs} Y_1^s + V_1^r B_{12}^{rs} Y_2^s + V_1^r B_{11}^{rr} Y_1^r + V_1^r B_{12}^{rr} Y_2^r + V_1^r B_{11}^{rt} Y_1^t + V_1^r B_{12}^{rt} Y_2^t - V_1^s L_{11}^{rr} Y_1^s - V_1^s L_{12}^{rr} Y_2^s \\ V_2^r B_{21}^{rs} Y_1^s + V_2^r B_{22}^{rs} Y_2^s + V_2^r B_{21}^{rr} Y_1^r + V_2^r B_{22}^{rr} Y_2^r + V_2^r B_{21}^{rt} Y_1^t + V_2^r B_{22}^{rt} Y_2^t - V_1^s L_{21}^{rr} Y_1^s - V_1^s L_{22}^{rr} Y_2^s \\ V_1^t B_{11}^{ts} Y_1^s + V_1^t B_{12}^{ts} Y_2^s + V_1^t B_{11}^{tr} Y_1^r + V_1^t B_{12}^{tr} Y_2^r + V_1^t B_{11}^{tt} Y_1^t + V_1^t B_{12}^{tt} Y_2^t - V_1^s L_{11}^{tt} Y_1^s - V_1^s L_{12}^{tt} Y_2^s \\ V_2^t B_{21}^{ts} Y_1^s + V_2^t B_{22}^{ts} Y_2^s + V_2^t B_{21}^{tr} Y_1^r + V_2^t B_{22}^{tr} Y_2^r + V_2^t B_{21}^{tt} Y_1^t + V_2^t B_{22}^{tt} Y_2^t - V_1^s L_{21}^{tt} Y_1^s - V_1^s L_{22}^{tt} Y_2^s \end{pmatrix}$$

$$\text{rowSums}(\widehat{\hat{V}}(B-L)\widehat{\text{rowSums}(Y)}) = \begin{pmatrix} V_1^s \left(\sum_{u=s,j=1}^{t,2} B_{1j}^{su} Y_j^u - \sum_{j=1}^2 L_{1j}^{ss} Y_j^s \right) \\ V_2^s \left(\sum_{u=s,j=1}^{t,2} B_{2j}^{su} Y_j^u - \sum_{j=1}^2 L_{2j}^{ss} Y_j^s \right) \\ V_1^r \left(\sum_{u=s,j=1}^{t,2} B_{1j}^{ru} Y_j^u - \sum_{j=1}^2 L_{1j}^{rr} Y_j^r \right) \\ V_2^r \left(\sum_{u=s,j=1}^{t,2} B_{2j}^{ru} Y_j^u - \sum_{j=1}^2 L_{2j}^{rr} Y_j^r \right) \\ V_1^t \left(\sum_{u=s,j=1}^{t,2} B_{1j}^{tu} Y_j^u - \sum_{j=1}^2 L_{1j}^{tt} Y_j^t \right) \\ V_2^t \left(\sum_{u=s,j=1}^{t,2} B_{2j}^{tu} Y_j^u - \sum_{j=1}^2 L_{2j}^{tt} Y_j^t \right) \end{pmatrix}$$

根据该结果不易确认目的地是增加值
吸收国还是直接进口国，因此根据公
式 $LAmB=B-L$ 对该矩阵作进一步变换

VA_GVC分解至国家_行业.世界——V(B-L)Y分解

以上矩阵表示方法不便确认分解思路，因此采用 $B-L=LA_mB$ 的等式进行替换

$$\begin{cases} AdX + AmX + Y = X \\ BY = X \end{cases} \quad AmX + Y = X - AdX \quad AmBY + Y = (I - Ad)X \quad LAmBY + LY = X \quad LAmBY + LY = BY \quad LAmB + L = B$$

$$L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} \quad B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} \quad Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$LAmB = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix} \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

VA_GVC分解至国家_行业.世界——V(B-L)Y分解

$$LAmB = \begin{pmatrix} 0 & 0 & \sum_{i=1}^2 L_{1i}^{ss} A_{i1}^{sr} & \sum_{i=1}^2 L_{1i}^{ss} A_{i2}^{sr} & \sum_{i=1}^2 L_{1i}^{ss} A_{i1}^{st} & \sum_{i=1}^2 L_{1i}^{ss} A_{i2}^{st} \\ 0 & 0 & \sum_{i=1}^2 L_{2i}^{ss} A_{i1}^{sr} & \sum_{i=1}^2 L_{2i}^{ss} A_{i2}^{sr} & \sum_{i=1}^2 L_{2i}^{ss} A_{i1}^{st} & \sum_{i=1}^2 L_{2i}^{ss} A_{i2}^{st} \\ \sum_{i=1}^2 L_{1i}^{rr} A_{i1}^{rs} & \sum_{i=1}^2 L_{1i}^{rr} A_{i2}^{rs} & 0 & 0 & \sum_{i=1}^2 L_{1i}^{rr} A_{i1}^{rt} & \sum_{i=1}^2 L_{1i}^{rr} A_{i2}^{rt} \\ \sum_{i=1}^2 L_{2i}^{rr} A_{i1}^{rs} & \sum_{i=1}^2 L_{2i}^{rr} A_{i2}^{rs} & 0 & 0 & \sum_{i=1}^2 L_{2i}^{rr} A_{i1}^{rt} & \sum_{i=1}^2 L_{2i}^{rr} A_{i2}^{rt} \\ \sum_{i=1}^2 L_{1i}^{tt} A_{i1}^{ts} & \sum_{i=1}^2 L_{1i}^{tt} A_{i2}^{ts} & \sum_{i=1}^2 L_{1i}^{tt} A_{i1}^{tr} & \sum_{i=1}^2 L_{1i}^{tt} A_{i2}^{tr} & 0 & 0 \\ \sum_{i=1}^2 L_{2i}^{tt} A_{i1}^{ts} & \sum_{i=1}^2 L_{2i}^{tt} A_{i2}^{ts} & \sum_{i=1}^2 L_{2i}^{tt} A_{i1}^{tr} & \sum_{i=1}^2 L_{2i}^{tt} A_{i2}^{tr} & 0 & 0 \end{pmatrix} \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$LAmB = \begin{pmatrix} \sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us}) & \sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us}) & \sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j1}^{ur}) & \sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ur}) & \sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j1}^{ut}) & \sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ut}) \\ \sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us}) & \sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us}) & \sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j1}^{ur}) & \sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ur}) & \sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j1}^{ut}) & \sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ut}) \\ \sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{us}) & \sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{us}) & \sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j1}^{ur}) & \sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur}) & \sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j1}^{ut}) & \sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ut}) \\ \sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{us}) & \sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{us}) & \sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j1}^{ur}) & \sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur}) & \sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j1}^{ut}) & \sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ut}) \\ \sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{us}) & \sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{us}) & \sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j1}^{ur}) & \sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ur}) & \sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j1}^{ut}) & \sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut}) \\ \sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{us}) & \sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{us}) & \sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j1}^{ur}) & \sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ur}) & \sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j1}^{ut}) & \sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut}) \end{pmatrix}$$

VA_GVC分解至国家_行业.世界——V(B-L)Y分解

$$B - L = \begin{pmatrix} B_{11}^{ss} - L_{11}^{ss} & B_{12}^{ss} - L_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} - L_{21}^{ss} & B_{22}^{ss} - L_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} - L_{11}^{rr} & B_{12}^{rr} - L_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} - L_{21}^{rr} & B_{22}^{rr} - L_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} - L_{11}^{tt} & B_{12}^{tt} - L_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} - L_{21}^{tt} & B_{22}^{tt} - L_{22}^{tt} \end{pmatrix}$$

$$LAmB = \begin{pmatrix} \sum_{i=1}^2 L_{1i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us} \right) & \sum_{i=1}^2 L_{1i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us} \right) & \sum_{i=1}^2 L_{1i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ur} \right) & \sum_{i=1}^2 L_{1i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ur} \right) & \sum_{i=1}^2 L_{1i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ut} \right) & \sum_{i=1}^2 L_{1i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ut} \right) \\ \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us} \right) & \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us} \right) & \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ur} \right) & \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ur} \right) & \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ut} \right) & \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ut} \right) \\ \sum_{i=1}^2 L_{1i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{us} \right) & \sum_{i=1}^2 L_{1i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{us} \right) & \sum_{i=1}^2 L_{1i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ur} \right) & \sum_{i=1}^2 L_{1i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur} \right) & \sum_{i=1}^2 L_{1i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ut} \right) & \sum_{i=1}^2 L_{1i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ut} \right) \\ \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{us} \right) & \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{us} \right) & \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ur} \right) & \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur} \right) & \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ut} \right) & \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ut} \right) \\ \sum_{i=1}^2 L_{1i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{us} \right) & \sum_{i=1}^2 L_{1i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{us} \right) & \sum_{i=1}^2 L_{1i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ur} \right) & \sum_{i=1}^2 L_{1i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ur} \right) & \sum_{i=1}^2 L_{1i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ut} \right) & \sum_{i=1}^2 L_{1i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut} \right) \\ \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{us} \right) & \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{us} \right) & \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ur} \right) & \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ur} \right) & \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ut} \right) & \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut} \right) \end{pmatrix}$$

VA_GVC分解至国家_行业.世界——V(B-L)Y分解

$$\text{rowSums}(\widehat{V}(B-L)\widehat{\text{rowSums}}(Y)) = \begin{pmatrix} V_1^s B_{11}^{ss} Y_1^s + V_1^s B_{12}^{ss} Y_2^s + V_1^s B_{11}^{sr} Y_1^r + V_1^s B_{12}^{sr} Y_2^r + V_1^s B_{11}^{st} Y_1^t + V_1^s B_{12}^{st} Y_2^t - V_1^s L_{11}^{ss} Y_1^s - V_1^s L_{12}^{ss} Y_1^s \\ V_2^s B_{21}^{ss} Y_1^s + V_2^s B_{22}^{ss} Y_2^s + V_2^s B_{21}^{sr} Y_1^r + V_2^s B_{22}^{sr} Y_2^r + V_2^s B_{21}^{st} Y_1^t + V_2^s B_{22}^{st} Y_2^t - V_1^s L_{21}^{ss} Y_1^s - V_1^s L_{22}^{ss} Y_1^s \\ V_1^r B_{11}^{rs} Y_1^s + V_1^r B_{12}^{rs} Y_2^s + V_1^r B_{11}^{rr} Y_1^r + V_1^r B_{12}^{rr} Y_2^r + V_1^r B_{11}^{rt} Y_1^t + V_1^r B_{12}^{rt} Y_2^t - V_1^s L_{11}^{rr} Y_1^s - V_1^s L_{12}^{rr} Y_1^s \\ V_2^r B_{21}^{rs} Y_1^s + V_2^r B_{22}^{rs} Y_2^s + V_2^r B_{21}^{rr} Y_1^r + V_2^r B_{22}^{rr} Y_2^r + V_2^r B_{21}^{rt} Y_1^t + V_2^r B_{22}^{rt} Y_2^t - V_1^s L_{21}^{rr} Y_1^s - V_1^s L_{22}^{rr} Y_1^s \\ V_1^t B_{11}^{ts} Y_1^s + V_1^t B_{12}^{ts} Y_2^s + V_1^t B_{11}^{tr} Y_1^r + V_1^t B_{12}^{tr} Y_2^r + V_1^t B_{11}^{tt} Y_1^t + V_1^t B_{12}^{tt} Y_2^t - V_1^s L_{11}^{tt} Y_1^s - V_1^s L_{12}^{tt} Y_1^s \\ V_2^t B_{21}^{ts} Y_1^s + V_2^t B_{22}^{ts} Y_2^s + V_2^t B_{21}^{tr} Y_1^r + V_2^t B_{22}^{tr} Y_2^r + V_2^t B_{21}^{tt} Y_1^t + V_2^t B_{22}^{tt} Y_2^t - V_1^s L_{21}^{tt} Y_1^s - V_1^s L_{22}^{tt} Y_1^s \end{pmatrix}$$

$$\text{rowSums}(\widehat{V}(B-L)\widehat{\text{rowSums}}(Y)) = \text{rowSums}(\widehat{V}L\text{Am}BY)$$

$$\begin{pmatrix} V_1^s [\sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us})] Y_1^s + V_1^s [\sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us})] Y_2^s + V_1^s [\sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j1}^{ur})] Y_1^r + V_1^s [\sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ur})] Y_2^r + V_1^s [\sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j1}^{ut})] Y_1^t + V_1^s [\sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ut})] Y_2^t \\ V_2^s [\sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us})] Y_1^s + V_2^s [\sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us})] Y_2^s + V_2^s [\sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j1}^{ur})] Y_1^r + V_2^s [\sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ur})] Y_2^r + V_2^s [\sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j1}^{ut})] Y_1^t + V_2^s [\sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ut})] Y_2^t \\ V_1^r [\sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{us})] Y_1^s + V_1^r [\sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{us})] Y_2^s + V_1^r [\sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j1}^{ur})] Y_1^r + V_1^r [\sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur})] Y_2^r + V_1^r [\sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j1}^{ut})] Y_1^t + V_1^r [\sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ut})] Y_2^t \\ V_2^r [\sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{us})] Y_1^s + V_2^r [\sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{us})] Y_2^s + V_2^r [\sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j1}^{ur})] Y_1^r + V_2^r [\sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur})] Y_2^r + V_2^r [\sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j1}^{ut})] Y_1^t + V_2^r [\sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ut})] Y_2^t \\ V_1^t [\sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{us})] Y_1^s + V_1^t [\sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{us})] Y_2^s + V_1^t [\sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j1}^{ur})] Y_1^r + V_1^t [\sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ur})] Y_2^r + V_1^t [\sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j1}^{ut})] Y_1^t + V_1^t [\sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut})] Y_2^t \\ V_2^t [\sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{us})] Y_1^s + V_2^t [\sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{us})] Y_2^s + V_2^t [\sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j1}^{ur})] Y_1^r + V_2^t [\sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ur})] Y_2^r + V_2^t [\sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j1}^{ut})] Y_1^t + V_2^t [\sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut})] Y_2^t \end{pmatrix}$$





VA_GVC分解至国家_行业.世界——V(B-L)Y分解

$$\text{rowSums}(\widehat{V}(\widehat{B-L})\widehat{\text{rowSums}}(Y)) = \text{rowSums}(\widehat{V}L\widehat{A}mBY)$$

$$= \begin{pmatrix} V_1^s L_{11}^{ss} \sum_{u \neq s, j=1}^{t,2} A_{1j}^{su} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) + V_1^s L_{12}^{ss} \sum_{u \neq s, j=1}^{t,2} A_{2j}^{su} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) \\ V_2^s L_{21}^{ss} \sum_{u \neq s, j=1}^{t,2} A_{1j}^{su} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) + V_2^s L_{22}^{ss} \sum_{u \neq s, j=1}^{t,2} A_{2j}^{su} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) \\ V_1^r L_{11}^{rr} \sum_{u \neq r, j=1}^{t,2} A_{1j}^{ru} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) + V_1^r L_{12}^{rr} \sum_{u \neq r, j=1}^{t,2} A_{2j}^{ru} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) \\ V_2^r L_{21}^{rr} \sum_{u \neq r, j=1}^{t,2} A_{1j}^{ru} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) + V_2^r L_{22}^{rr} \sum_{u \neq r, j=1}^{t,2} A_{2j}^{ru} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) \\ V_1^t L_{11}^{tt} \sum_{u \neq t, j=1}^{r,2} A_{1j}^{tu} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) + V_1^t L_{12}^{tt} \sum_{u \neq t, j=1}^{r,2} A_{2j}^{tu} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) \\ V_2^t L_{21}^{tt} \sum_{u \neq t, j=1}^{r,2} A_{1j}^{tu} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) + V_2^t L_{22}^{tt} \sum_{u \neq t, j=1}^{r,2} A_{2j}^{tu} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) \end{pmatrix}$$

分解思路:

s国向r国出口中间品并参与GVC生产活动中包含的s国部门2的增加值+s国向t国出口中间品并参与GVC生产活动中包含的s国部门2的增加值

分解思路解析:

1.目的地确认: 以直接进口国作为目的地

2.理由:

首先,要明确参与GVC生产活动的方式。参与GVC生产活动主要是通过中间品贸易(进口与出口)实现,其中包括四种形式:(1)中间产品出口,直接进口国用来生产自己最终吸收的产品;(2)中间产品出口,直接进口国用来生产出口的产品;(3)中间产品进口,生产的产品供自己最终吸收;(4)中间产品进口,生产的产品用来出口。

其次,根据WWZ(2015)的贸易分解法可知,在双边出口分解中,例如s国对r国出口分解时,直接进口国r作为出口目的地,并非以增加值最终吸收地作为目的地(在RDV中,最终吸收地是s国,出口目的地是r国;又如DVA_INTre的最终吸收地是第三国,但是出口目的地依旧是r国)。

根据以上理由,GVC生产活动的双边分解中以直接进口国作为目的地。

二：Xvd_GVC(即PLv_GVC分子)分解至国家_行业.世界——简单与复杂

UIBE的代码修改：

```
VLL<-diag(VC)%**%L%**%L
```

#简单GVC生产活动引致的国内总产出

```
Xvd_GVC_S<-rowSums(VLL%**%Am%**%L%**%diag(rowSums(Yd)))
```

#复杂GVC生产活动引致的国内总产出

```
z1 <- Am%**%B%**%diag(rowSums( Y ) )
```

```
zm <- z1
```

```
for (s in 1:G){
```

```
  m <- 1+( s-1 )*N; n <- N+( s-1 )*N
```

```
  zm[ m:n,m:n ] <- 0
```

```
}
```

```
zd <- z1 - zm
```

#以下是复杂GVC引致的国内总产出的分解

```
Xvd_GVC_D <- VLL%**%rowSums(zd)
```

```
Xvd_GVC_FR <-VLL%**%rowSums(zm)
```

```
Xvd_GVC_F <-Xvd_GVC_FR-Xvd_GVC_S
```

#GVC引致的国内总产出

```
Xvd_GVC<-VLL%**%rowSums(z1)
```

等价于：Xvd_GVC<-Xvd_GVC_S+Xvd_GVC_D+Xvd_GVC_F

Xvd_GVC分解至国家_行业.世界——复杂+简单GVC引致的国内总产出

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} \quad L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} \quad \hat{V} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix} = \begin{pmatrix} \sum_{u=s}^t Y_1^{su} \\ \sum_{u=s}^t Y_2^{su} \\ \sum_{u=s}^t Y_1^{ru} \\ \sum_{u=s}^t Y_2^{ru} \\ \sum_{u=s}^t Y_1^{tu} \\ \sum_{u=s}^t Y_2^{tu} \end{pmatrix} \quad \widehat{\text{rowSums}(Y)} = \begin{pmatrix} Y_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_2^t \end{pmatrix} = \begin{pmatrix} \sum_{u=s}^t Y_1^{su} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sum_{u=s}^t Y_2^{su} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{u=s}^t Y_1^{ru} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sum_{u=s}^t Y_2^{ru} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{u=s}^t Y_1^{tu} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sum_{u=s}^t Y_2^{tu} \end{pmatrix}$$

Xvd_GVC分解至国家_行业.世界——复杂+简单GVC引致的国内总产出

$$LL = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix}$$

$$LL = \begin{pmatrix} \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \end{pmatrix}$$

$$Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$AmB = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix} \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$\text{Am}(\overbrace{\text{rowSums}(\mathbf{Y})}) = \begin{pmatrix} \sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} B_{jl}^{ks} & \sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} B_{j2}^{ks} & \sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} B_{jl}^{kr} & \sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} B_{j2}^{kr} & \sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} B_{jl}^{kt} & \sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} B_{j2}^{kt} \\ \sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} B_{jl}^{ks} & \sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} B_{j2}^{ks} & \sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} B_{jl}^{kr} & \sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} B_{j2}^{kr} & \sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} B_{jl}^{kt} & \sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} B_{j2}^{kt} \\ \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} B_{jl}^{ks} & \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} B_{j2}^{ks} & \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} B_{jl}^{kr} & \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} B_{j2}^{kr} & \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} B_{jl}^{kt} & \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} B_{j2}^{kt} \\ \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} B_{jl}^{ks} & \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} B_{j2}^{ks} & \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} B_{jl}^{kr} & \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} B_{j2}^{kr} & \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} B_{jl}^{kt} & \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} B_{j2}^{kt} \\ \sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} B_{jl}^{ks} & \sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} B_{j2}^{ks} & \sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} B_{jl}^{kr} & \sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} B_{j2}^{kr} & \sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} B_{jl}^{kt} & \sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} B_{j2}^{kt} \\ \sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} B_{jl}^{ks} & \sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} B_{j2}^{ks} & \sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} B_{jl}^{kr} & \sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} B_{j2}^{kr} & \sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} B_{jl}^{kt} & \sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} B_{j2}^{kt} \end{pmatrix} \begin{pmatrix} \mathbf{Y}_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{Y}_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{Y}_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{Y}_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{Y}_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{Y}_2^t \end{pmatrix}$$

Xvd_GVC分解至国家_行业.世界——复杂+简单GVC引致的国内总产出

$$\text{AmB}(\text{rowSums}(Y)) = \begin{pmatrix} \sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} B_{jl}^{ks} Y_1^s & \sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} B_{j2}^{ks} Y_2^s & \sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} B_{jl}^{kr} Y_1^r & \sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} B_{j2}^{kr} Y_2^r & \sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} B_{jl}^{kt} Y_1^t & \sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} B_{j2}^{kt} Y_2^t \\ \sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} B_{jl}^{ks} Y_1^s & \sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} B_{j2}^{ks} Y_2^s & \sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} B_{jl}^{kr} Y_1^r & \sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} B_{j2}^{kr} Y_2^r & \sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} B_{jl}^{kt} Y_1^t & \sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} B_{j2}^{kt} Y_2^t \\ \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} B_{jl}^{ks} Y_1^s & \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} B_{j2}^{ks} Y_2^s & \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} B_{jl}^{kr} Y_1^r & \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} B_{j2}^{kr} Y_2^r & \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} B_{jl}^{kt} Y_1^t & \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} B_{j2}^{kt} Y_2^t \\ \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} B_{jl}^{ks} Y_1^s & \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} B_{j2}^{ks} Y_2^s & \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} B_{jl}^{kr} Y_1^r & \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} B_{j2}^{kr} Y_2^r & \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} B_{jl}^{kt} Y_1^t & \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} B_{j2}^{kt} Y_2^t \\ \sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} B_{jl}^{ks} Y_1^s & \sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} B_{j2}^{ks} Y_2^s & \sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} B_{jl}^{kr} Y_1^r & \sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} B_{j2}^{kr} Y_2^r & \sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} B_{jl}^{kt} Y_1^t & \sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} B_{j2}^{kt} Y_2^t \\ \sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} B_{jl}^{ks} Y_1^s & \sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} B_{j2}^{ks} Y_2^s & \sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} B_{jl}^{kr} Y_1^r & \sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} B_{j2}^{kr} Y_2^r & \sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} B_{jl}^{kt} Y_1^t & \sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} B_{j2}^{kt} Y_2^t \end{pmatrix}$$

$$\text{rowSums}(\text{AmB}(\text{rowSums}(Y))) = \begin{pmatrix} \sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \\ \sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \\ \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \\ \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \\ \sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \\ \sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \end{pmatrix}$$

Xvd_GVC分解至国家_行业.世界——复杂+简单GVC引致的国内总产出

$$\hat{V} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix}$$

$$\hat{V}_{LL} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \begin{pmatrix} \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \end{pmatrix}$$

$$\hat{V}_{LL} = \begin{pmatrix} V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} & V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} & V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} & V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} & V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} & V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \\ 0 & 0 & 0 & 0 & V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} & V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \end{pmatrix}$$

Xvd_GVC分解至国家_行业.世界——复杂+简单GVC引致的国内总产出

$$\widehat{VLL}rowSums(AmB(\widehat{rowSums(Y)})) = \begin{pmatrix} V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} & V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} & V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} & V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} & V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} & V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \\ 0 & 0 & 0 & 0 & V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} & V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \end{pmatrix} \begin{pmatrix} \sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \\ \sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \\ \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \\ \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \\ \sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \\ \sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \end{pmatrix}$$

Xvd_GVC分解至国家_行业.世界——复杂+简单GVC引致的国内总产出

$$\widehat{VLLrowSums}(AmB(\widehat{rowSums}(Y))) = Xvd_GVC = \begin{pmatrix} V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} \left[\sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \right] + V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} \left[\sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \right] \\ V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} \left[\sum_{k \neq s, j=1}^{t,2} A_{1j}^{sk} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \right] + V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} \left[\sum_{k \neq s, j=1}^{t,2} A_{2j}^{sk} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \right] \\ V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} \left[\sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \right] + V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} \left[\sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \right] \\ V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} \left[\sum_{k \neq r, j=1}^{t,2} A_{1j}^{rk} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \right] + V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} \left[\sum_{k \neq r, j=1}^{t,2} A_{2j}^{rk} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \right] \\ V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} \left[\sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \right] + V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \left[\sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \right] \\ \boxed{V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} \left[\sum_{k \neq t, j=1}^{r,2} A_{1j}^{tk} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \right] + V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \left[\sum_{k \neq t, j=1}^{r,2} A_{2j}^{tk} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ku} Y_i^u \right) \right]} \end{pmatrix}$$

分解思路解析：t国向s国出口中间品中包含的t国部门2的增加值的生产活动中引致的国内总产出+t国向r国出口中间品中包含的t国部门2的增加值的生产活动中引致的国内总产出

三：Xvd_GVC(即PLv_GVC分子)分解至国家_行业.目的地——简单与复杂

```
VLLAmBY分解至国家_行业.国家的代码： 即Xvd_GVC
VLLAmBY_forward<-array(0,dim=c(GN*G,1))
VLL<-diag(VC)%*%L%*%L
Y_1<-array(rowSums(Y),dim=c(GN,GN))
BY<-t(B%*%Y_1)
AmBY<-Am*BY
for (i in 1:G) {
  m=1+(i-1)*N ; n=N+(i-1)*N
  s=1+(i-1)*GN ; u=GN+(i-1)*GN
  AmBY_1<-array(rowSums(AmBY[,m:n]),dim=c(GN,GN))
  AmBY_2<-t(AmBY_1)
  AmBY_F<-AmBY_2
  for (j in 1:G) {
    mm=1+(j-1)*N ; nn=N+(j-1)*N
    AmBY_F[mm:nn,mm:nn]<-0
  }
  AmBY_D<-AmBY_2-AmBY_F
  VLLAmBY_1<-VLL*AmBY_D
  VLLAmBY_2<-as.matrix(rowSums(VLLAmBY_1))
  VLLAmBY_forward[s:u,1]<-VLLAmBY_2[,1]
}
```

Xvd_GVC分解至国家_行业.目的地——VLLAmBY

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix}$$

$$\overline{\text{rowSums}(Y)} = \begin{pmatrix} Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \end{pmatrix}$$

$$Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$\overline{\text{BrowSums}(Y)} =$$

$$\begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} \begin{pmatrix} Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \end{pmatrix}$$

$$\overline{\text{BrowSums}(Y)} =$$

$$\begin{pmatrix} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u \\ \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u \\ \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u \\ \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u \\ \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u \\ \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u & \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \end{pmatrix}$$

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$$\begin{aligned} \overline{\text{(BrowSums(Y))}}^T &= \left(\sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u \quad \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u \quad \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u \quad \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u \quad \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u \quad \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \right) \\ Am * \overline{\text{(BrowSums(Y))}}^T &= \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix} * \left(\sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u \quad \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u \quad \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u \quad \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u \quad \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u \quad \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \right) \end{aligned}$$

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$$\text{Am} * (\overline{\text{BrowSums}(Y)})^T = \begin{pmatrix} 0 & 0 & A_{11}^{sr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & A_{12}^{sr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & A_{11}^{st} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & A_{12}^{st} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ 0 & 0 & A_{21}^{sr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & A_{22}^{sr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & A_{21}^{st} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & A_{22}^{st} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ A_{11}^{rs} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & A_{12}^{rs} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & 0 & 0 & A_{11}^{rt} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & A_{12}^{rt} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ A_{21}^{rs} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & A_{22}^{rs} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & 0 & 0 & A_{21}^{rt} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & A_{22}^{rt} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ A_{11}^{ts} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & A_{12}^{ts} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & A_{11}^{tr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & A_{12}^{tr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & 0 & 0 \\ A_{21}^{ts} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & A_{22}^{ts} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & A_{21}^{tr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & A_{22}^{tr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & 0 & 0 \end{pmatrix}$$

$$\text{rowSums}(\text{Am} * (\overline{\text{BrowSums}(Y)})^T [1:2]) = \begin{pmatrix} 0 \\ 0 \\ A_{11}^{rs} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u + A_{12}^{rs} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u \\ A_{21}^{rs} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u + A_{22}^{rs} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u \\ A_{11}^{ts} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u + A_{12}^{ts} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u \\ A_{21}^{ts} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u + A_{22}^{ts} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \\ \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \\ \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \\ \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \end{pmatrix}$$

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[illegible]

$$\overline{(\text{rowSums}(\text{Am} * (\text{BrowSums}(\mathbf{Y}))^T[1:2]))^T} = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{u=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{u=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{u=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{u=1}^{t,2} B_{ji}^{su} Y_i^u \right) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \end{pmatrix}$$

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$$\overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [1:2]))^T)^F} = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & 0 & 0 \end{pmatrix}$$

$$\overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y})) [1:2])))} = \overline{(\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [1:2]))} - \overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y})) [1:2])))}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \end{pmatrix}$$

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$$\widehat{VLL} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [1:2]))^T)^D$$

$$= \begin{pmatrix} V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} & V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} & V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} & V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} & V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} & V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \\ 0 & 0 & 0 & 0 & V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} & V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \end{pmatrix} * \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) \end{pmatrix}$$

$$\widehat{VLL} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [1:2]))^T)^D =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rs} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) & V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rs} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) & 0 & 0 \\ 0 & 0 & V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} \sum_{k \neq r, j=1}^{t,2} A_{1j}^{rs} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) & V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} \sum_{k \neq r, j=1}^{t,2} A_{2j}^{rs} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} \sum_{k \neq t, j=1}^{r,2} A_{1j}^{ts} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) & V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \sum_{k \neq t, j=1}^{r,2} A_{2j}^{ts} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) \\ 0 & 0 & 0 & 0 & V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} \sum_{k \neq t, j=1}^{r,2} A_{1j}^{ts} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) & V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \sum_{k \neq t, j=1}^{r,2} A_{2j}^{ts} (\sum_{u=s,i=1}^{t,2} B_{ji}^{su} Y_i^u) \end{pmatrix}$$

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$$\text{rowSums}(\widehat{\text{VLL}} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [1:2]))^T)^D) =$$

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$$\begin{pmatrix} 0 \\ 0 \\ V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} \left[\sum_{k \neq r, j=1}^{t,2} A_{1j}^{rs} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \right] + V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} \left[\sum_{k \neq r, j=1}^{t,2} A_{2j}^{rs} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \right] \\ V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} \left[\sum_{k \neq r, j=1}^{t,2} A_{1j}^{rs} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \right] + V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} \left[\sum_{k \neq r, j=1}^{t,2} A_{2j}^{rs} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \right] \\ V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} \left[\sum_{k \neq t, j=1}^{r,2} A_{1j}^{ts} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \right] + V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \left[\sum_{k \neq t, j=1}^{r,2} A_{2j}^{ts} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \right] \\ V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} \left[\sum_{k \neq t, j=1}^{r,2} A_{1j}^{ts} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \right] + V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \left[\sum_{k \neq t, j=1}^{r,2} A_{2j}^{ts} \left(\sum_{u=s, i=1}^{t,2} B_{ji}^{su} Y_i^u \right) \right] \end{pmatrix}$$

t国向s国出口中间品中包含
的t国部门2的增加值的生产
活动中引致的国内总产出

Xvd_GVC分解至国家_行业.目的地——VLLAmBY

$$\text{Am} * (\overline{\text{BrowSums}(Y)})^T = \begin{pmatrix} 0 & 0 & A_{11}^{sr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & A_{12}^{sr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & A_{11}^{st} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & A_{12}^{st} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ 0 & 0 & A_{21}^{sr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & A_{22}^{sr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & A_{21}^{st} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & A_{22}^{st} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ A_{11}^{rs} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & A_{12}^{rs} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & 0 & 0 & A_{11}^{rt} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & A_{12}^{rt} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ A_{21}^{rs} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & A_{22}^{rs} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & 0 & 0 & A_{21}^{rt} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & A_{22}^{rt} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ A_{11}^{ts} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & A_{12}^{ts} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & A_{11}^{tr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & A_{12}^{tr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & 0 & 0 \\ A_{21}^{ts} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & A_{22}^{ts} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & A_{21}^{tr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & A_{22}^{tr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & 0 & 0 \end{pmatrix}$$

$$\text{rowSums}(\text{Am} * (\overline{\text{BrowSums}(Y)})^T [3:4]) = \begin{pmatrix} A_{11}^{sr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u + A_{12}^{sr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u \\ A_{21}^{sr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u + A_{22}^{sr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u \\ 0 \\ 0 \\ A_{11}^{tr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u + A_{12}^{tr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u \\ A_{21}^{tr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u + A_{22}^{tr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{sr} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u \right) \\ \sum_{j=1}^2 A_{2j}^{sr} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u \right) \\ 0 \\ 0 \\ \sum_{j=1}^2 A_{1j}^{tr} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u \right) \\ \sum_{j=1}^2 A_{2j}^{tr} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u \right) \end{pmatrix}$$

Xvd_GVC分解至国家_行业.目的地——VLLAmBY

[illegible]

$$\overline{(\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[3:4]))^T} = \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) \end{pmatrix}$$

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$$\overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[3:4]))^T)^F} = \begin{pmatrix} 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{\text{tr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{\text{tr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{\text{tr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{\text{tr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) \\ \sum_{j=1}^2 A_{1j}^{\text{sr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{\text{sr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{\text{tr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{\text{tr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) \\ \sum_{j=1}^2 A_{1j}^{\text{sr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{\text{sr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{\text{tr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{\text{tr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) \\ \sum_{j=1}^2 A_{1j}^{\text{sr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{\text{sr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{\text{sr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{\text{sr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[3:4]))^T)^D} = \overline{(\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[3:4]))^T} - \overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[3:4]))^T)^F}$$

$$= \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{\text{sr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{\text{sr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{\text{sr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{\text{sr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{\text{tr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{\text{tr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{\text{tr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) & \sum_{j=1}^2 A_{2j}^{\text{tr}} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{\text{ru}} Y_i^u \right) \end{pmatrix}$$

Xvd_GVC分解至国家_行业.目的地——VLLAmBY

$$\widehat{VLL} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [, 3 : 4]))^T)^D$$

$$= \begin{pmatrix} V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} & V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} & V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} & V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} & V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} & V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \\ 0 & 0 & 0 & 0 & V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} & V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \end{pmatrix} * \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & 0 & 0 \end{pmatrix}$$

$$\widehat{VLL} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [, 3 : 4]))^T)^D =$$

$$\begin{pmatrix} V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & 0 & 0 & 0 & 0 \\ V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} \sum_{j=1}^2 A_{1j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \sum_{j=1}^2 A_{2j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & 0 & 0 \\ 0 & 0 & V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} \sum_{j=1}^2 A_{1j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \sum_{j=1}^2 A_{2j}^{tr} (\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u) & 0 & 0 \end{pmatrix}$$

Xvd_GVC分解至国家_行业.目的地——VLLAmBY

$$\text{rowSums}(\widehat{\text{VLL}} * ((\text{rowSums}(\text{Am} * (\overline{\text{BrowSums}(\text{Y}))^T [3:4]))^T)^D) =$$

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$$\begin{pmatrix} V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} \left[\sum_{j=1}^2 A_{1j}^{sr} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u \right) \right] + V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} \left[\sum_{j=1}^2 A_{2j}^{sr} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u \right) \right] \\ V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} \left[\sum_{j=1}^2 A_{1j}^{sr} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u \right) \right] + V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} \left[\sum_{j=1}^2 A_{2j}^{sr} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u \right) \right] \\ 0 \\ 0 \\ V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} \left[\sum_{j=1}^2 A_{1j}^{tr} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u \right) \right] + V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \left[\sum_{j=1}^2 A_{2j}^{tr} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u \right) \right] \\ V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} \left[\sum_{j=1}^2 A_{1j}^{tr} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u \right) \right] + V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \left[\sum_{j=1}^2 A_{2j}^{tr} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{ru} Y_i^u \right) \right] \end{pmatrix}$$

t国向r国出口中间品中包含
的t国部门2的增加值的生产
活动中引致的国内总产出

Xvd_GVC分解至国家_行业.目的地——VLLAmBY

$$\text{Am} * (\overline{\text{BrowSums}(Y)})^T = \begin{pmatrix} 0 & 0 & A_{11}^{sr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & A_{12}^{sr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & A_{11}^{st} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & A_{12}^{st} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ 0 & 0 & A_{21}^{sr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & A_{22}^{sr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & A_{21}^{st} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & A_{22}^{st} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ A_{11}^{rs} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & A_{12}^{rs} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & 0 & 0 & A_{11}^{rt} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & A_{12}^{rt} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ A_{21}^{rs} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & A_{22}^{rs} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & 0 & 0 & A_{21}^{rt} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u & A_{22}^{rt} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ A_{11}^{ts} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & A_{12}^{ts} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & A_{11}^{tr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & A_{12}^{tr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & 0 & 0 \\ A_{21}^{ts} \sum_{u=s,i=1}^{t,2} B_{li}^{su} Y_i^u & A_{22}^{ts} \sum_{u=s,i=1}^{t,2} B_{2i}^{su} Y_i^u & A_{21}^{tr} \sum_{u=s,i=1}^{t,2} B_{li}^{ru} Y_i^u & A_{22}^{tr} \sum_{u=s,i=1}^{t,2} B_{2i}^{ru} Y_i^u & 0 & 0 \end{pmatrix}$$

$$\text{rowSums}(\text{Am} * (\overline{\text{BrowSums}(Y)})^T [5:6]) = \begin{pmatrix} A_{11}^{st} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u + A_{12}^{st} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ A_{21}^{st} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u + A_{22}^{st} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ A_{11}^{rt} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u + A_{12}^{rt} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ A_{21}^{rt} \sum_{u=s,i=1}^{t,2} B_{li}^{tu} Y_i^u + A_{22}^{rt} \sum_{u=s,i=1}^{t,2} B_{2i}^{tu} Y_i^u \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{st} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u \right) \\ \sum_{j=1}^2 A_{2j}^{st} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u \right) \\ \sum_{j=1}^2 A_{1j}^{rt} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u \right) \\ \sum_{j=1}^2 A_{2j}^{rt} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u \right) \\ 0 \\ 0 \end{pmatrix}$$

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$$\overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[5:6]))^T)^F} = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 \end{pmatrix}$$

$$\overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[5:6]))^T)^D} = \overline{(\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[5:6]))^T} - \overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[5:6]))^T)^F}$$

$$= \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Xvd_GVC分解至国家_行业.目的地——VLLAmBY

$$\widehat{VLL} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [5:6]))^T)^D$$

$$= \begin{pmatrix} V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} & V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} & V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} & V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} & V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} & V_1^t \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \\ 0 & 0 & 0 & 0 & V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} & V_2^t \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \end{pmatrix} * \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\widehat{VLL} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [5:6]))^T)^D =$$

$$\begin{pmatrix} V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} \sum_{j=1}^2 A_{1j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} \sum_{j=1}^2 A_{2j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 & 0 & 0 \\ V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} \sum_{j=1}^2 A_{1j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} \sum_{j=1}^2 A_{2j}^{st} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} \sum_{j=1}^2 A_{1j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} \sum_{j=1}^2 A_{2j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 \\ 0 & 0 & V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} \sum_{j=1}^2 A_{1j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} \sum_{j=1}^2 A_{2j}^{rt} (\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Xvd_GVC分解至国家_行业.目的地——VLLAmBY

$$\text{rowSums}(\widehat{\text{VLL}} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [5:6]))^T)^D) =$$

此矩阵的目的
地均为t国

$$\begin{pmatrix} V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} \left[\sum_{j=1}^2 A_{1j}^{st} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u \right) \right] + V_1^s \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} \left[\sum_{j=1}^2 A_{2j}^{st} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u \right) \right] \\ V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} \left[\sum_{j=1}^2 A_{1j}^{st} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u \right) \right] + V_2^s \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} \left[\sum_{j=1}^2 A_{2j}^{st} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u \right) \right] \\ V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} \left[\sum_{j=1}^2 A_{1j}^{rt} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u \right) \right] + V_1^r \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} \left[\sum_{j=1}^2 A_{2j}^{rt} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u \right) \right] \\ V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} \left[\sum_{j=1}^2 A_{1j}^{rt} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u \right) \right] + V_2^r \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} \left[\sum_{j=1}^2 A_{2j}^{rt} \left(\sum_{u=s,i=1}^{t,2} B_{ji}^{tu} Y_i^u \right) \right] \\ 0 \\ 0 \end{pmatrix}$$

r国向t国出口中间品中包含
的r国部门2的增加值的生产
活动中引致的国内总产出

四：Xvi_GVC(即PLv_GVC分子)分解至国家_行业.世界——简单与复杂

UIBE的代码修改：

#简单GVC生产活动引致的国际总产出

```
Xvi_GVC_S<-rowSums(diag(VC)%*%L%*%Am%*%L%*%L%*%diag(rowSums(Yd)))
```

#复杂GVC生产活动引致的国际总产出

```
z2 <- Am%*%B%*%B%*%diag(rowSums( Y ) )
```

```
zbm <- z2
```

```
for (s in 1:G){
```

```
  m <- 1+( s-1 )*N; n <- N+( s-1 )*N
```

```
  zbm[ m:n,m:n ] <- 0
```

```
}
```

```
zbd <- z2 -zbm
```

```
Xvi_GVC_D<-rowSums(diag( VC)%*%L%*%rowSums(zbd))
```

```
Xvi_GVC_FR<-rowSums(diag( VC)%*%L%*%(rowSums(zbm)))
```

```
Xvi_GVC_F<-Xvi_GVC_FR-Xvi_GVC_S
```

#GVC生产活动引致的国际总产出

```
Xvi_GVC_world<-rowSums(diag(VC)%*%L%*%rowSums(z2))
```

```
Xvi_GVC_world<-rowSums(diag(VC)%*%L%*%rowSums(Am%*%B%*%B%*%diag(rowSums( Y ) )))
```

等价于：Xvi_GVC_world<-Xvi_GVC_D+Xvi_GVC_F+Xvi_GVC_S

Xvi_GVC分解至国家_行业.世界——复杂+简单GVC引致的国际总产出

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} \quad L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} \quad \hat{V} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \quad Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix} = \begin{pmatrix} \sum_{u=s}^t Y_1^{su} \\ \sum_{u=s}^t Y_2^{su} \\ \sum_{u=s}^t Y_1^{ru} \\ \sum_{u=s}^t Y_2^{ru} \\ \sum_{u=s}^t Y_1^{tu} \\ \sum_{u=s}^t Y_2^{tu} \end{pmatrix} \quad \widehat{\text{rowSums}(Y)} = \begin{pmatrix} Y_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_2^t \end{pmatrix} = \begin{pmatrix} \sum_{u=s}^t Y_1^{su} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sum_{u=s}^t Y_2^{su} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{u=s}^t Y_1^{ru} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sum_{u=s}^t Y_2^{ru} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{u=s}^t Y_1^{tu} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sum_{u=s}^t Y_2^{tu} \end{pmatrix}$$

Xvi_GVC分解至国家_行业.世界——复杂+简单GVC引致的国际总产出

$$BB = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$\hat{V}L = \begin{pmatrix} V_1^s L_{11}^{ss} & V_1^s L_{12}^{ss} & 0 & 0 & 0 & 0 \\ V_2^s L_{21}^{ss} & V_2^s L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r L_{11}^{rr} & V_1^r L_{12}^{rr} & 0 & 0 \\ 0 & 0 & V_2^r L_{21}^{rr} & V_2^r L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t L_{11}^{tt} & V_1^t L_{12}^{tt} \\ 0 & 0 & 0 & 0 & V_2^t L_{21}^{tt} & V_2^t L_{22}^{tt} \end{pmatrix}$$

$$BB = \begin{pmatrix} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j1}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j1}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j1}^{ut} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j1}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j1}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j1}^{ut} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j1}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j1}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j1}^{ut} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j1}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j1}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j1}^{ut} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j1}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j1}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j1}^{ut} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j1}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j1}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j1}^{ut} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ut} \end{pmatrix}$$

Xvi_GVC分解至国家_行业.世界——复杂+简单GVC引致的国际总产出

$$\widehat{\text{BBrowSums}(Y)} = \begin{pmatrix} \sum_{u=s,j=1}^{t,2} B_{lj}^{su} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{lj}^{su} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{lj}^{su} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{lj}^{su} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{lj}^{su} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{lj}^{su} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{lj}^{ru} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{lj}^{ru} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{lj}^{ru} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{lj}^{ru} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{lj}^{ru} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{lj}^{ru} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{lj}^{tu} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{lj}^{tu} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{lj}^{tu} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{lj}^{tu} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{lj}^{tu} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{lj}^{tu} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ut} \end{pmatrix} \begin{pmatrix} Y_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_2^t \end{pmatrix}$$

$$\widehat{\text{BBrowSums}(Y)} = \begin{pmatrix} \sum_{u=s,j=1}^{t,2} B_{lj}^{su} B_{jl}^{us} Y_1^s & \sum_{u=s,j=1}^{t,2} B_{lj}^{su} B_{j2}^{us} Y_2^s & \sum_{u=s,j=1}^{t,2} B_{lj}^{su} B_{jl}^{ur} Y_1^r & \sum_{u=s,j=1}^{t,2} B_{lj}^{su} B_{j2}^{ur} Y_2^r & \sum_{u=s,j=1}^{t,2} B_{lj}^{su} B_{jl}^{ut} Y_1^t & \sum_{u=s,j=1}^{t,2} B_{lj}^{su} B_{j2}^{ut} Y_2^t \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{us} Y_1^s & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{us} Y_2^s & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ur} Y_1^r & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ur} Y_2^r & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ut} Y_1^t & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ut} Y_2^t \\ \sum_{u=s,j=1}^{t,2} B_{lj}^{ru} B_{jl}^{us} Y_1^s & \sum_{u=s,j=1}^{t,2} B_{lj}^{ru} B_{j2}^{us} Y_2^s & \sum_{u=s,j=1}^{t,2} B_{lj}^{ru} B_{jl}^{ur} Y_1^r & \sum_{u=s,j=1}^{t,2} B_{lj}^{ru} B_{j2}^{ur} Y_2^r & \sum_{u=s,j=1}^{t,2} B_{lj}^{ru} B_{jl}^{ut} Y_1^t & \sum_{u=s,j=1}^{t,2} B_{lj}^{ru} B_{j2}^{ut} Y_2^t \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{us} Y_1^s & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{us} Y_2^s & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ur} Y_1^r & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ur} Y_2^r & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ut} Y_1^t & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ut} Y_2^t \\ \sum_{u=s,j=1}^{t,2} B_{lj}^{tu} B_{jl}^{us} Y_1^s & \sum_{u=s,j=1}^{t,2} B_{lj}^{tu} B_{j2}^{us} Y_2^s & \sum_{u=s,j=1}^{t,2} B_{lj}^{tu} B_{jl}^{ur} Y_1^r & \sum_{u=s,j=1}^{t,2} B_{lj}^{tu} B_{j2}^{ur} Y_2^r & \sum_{u=s,j=1}^{t,2} B_{lj}^{tu} B_{jl}^{ut} Y_1^t & \sum_{u=s,j=1}^{t,2} B_{lj}^{tu} B_{j2}^{ut} Y_2^t \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{us} Y_1^s & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{us} Y_2^s & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ur} Y_1^r & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ur} Y_2^r & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ut} Y_1^t & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ut} Y_2^t \end{pmatrix}$$

[illegible]

Xvi_GVC分解至国家_行业.世界——复杂+简单GVC引致的国际总产出

$$\text{rowSums}(\text{AmBBrowSums}(Y)) =$$

$$\begin{pmatrix} \sum_{k \neq s, i=1}^{t,2} A_{li}^{sk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ \sum_{k \neq s, i=1}^{t,2} A_{2i}^{sk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ \sum_{k \neq s, i=1}^{t,2} A_{li}^{rk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ \sum_{k \neq s, i=1}^{t,2} A_{2i}^{rk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ \sum_{k \neq t, i=1}^{r,2} A_{li}^{tk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ \sum_{k \neq t, i=1}^{r,2} A_{2i}^{tk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \end{pmatrix}$$

$$\widehat{V} \text{LrowS}$$

$$\left(\begin{matrix} V_1^s L_{11}^{ss} & V_1^s L_{12}^{ss} & 0 & 0 & 0 & 0 \\ V_2^s L_{21}^{ss} & V_2^s L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r L_{11}^{rr} & V_1^r L_{12}^{rr} & 0 & 0 \\ 0 & 0 & V_2^r L_{21}^{rr} & V_2^r L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t L_{11}^{tt} & V_1^t L_{12}^{tt} \\ 0 & 0 & 0 & 0 & V_2^t L_{21}^{tt} & V_2^t L_{22}^{tt} \end{matrix} \right) =$$

$$\begin{pmatrix} \sum_{k \neq s, i=1}^{t,2} A_{li}^{sk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ \sum_{k \neq s, i=1}^{t,2} A_{2i}^{sk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ \sum_{k \neq s, i=1}^{t,2} A_{li}^{rk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ \sum_{k \neq s, i=1}^{t,2} A_{2i}^{rk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ \sum_{k \neq t, i=1}^{r,2} A_{li}^{tk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ \sum_{k \neq t, i=1}^{r,2} A_{2i}^{tk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \end{pmatrix}$$

Xvi_GVC分解至国家_行业.世界——复杂+简单GVC引致的国际总产出

$$\widehat{VLowSums}(AmBBrowSums(Y)) = \begin{pmatrix} V_1^s L_{11}^{ss} \sum_{k \neq s, i=1}^{t,2} A_{li}^{sk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_1^s L_{12}^{ss} \sum_{k \neq s, i=1}^{t,2} A_{2i}^{sk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ V_2^s L_{21}^{ss} \sum_{k \neq s, i=1}^{t,2} A_{li}^{sk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_2^s L_{22}^{ss} \sum_{k \neq s, i=1}^{t,2} A_{2i}^{sk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ V_1^r L_{11}^{rr} \sum_{k \neq s, i=1}^{t,2} A_{li}^{rk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_1^r L_{12}^{rr} \sum_{k \neq s, i=1}^{t,2} A_{2i}^{rk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ V_2^r L_{21}^{rr} \sum_{k \neq s, i=1}^{t,2} A_{li}^{rk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_2^r L_{22}^{rr} \sum_{k \neq s, i=1}^{t,2} A_{2i}^{rk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ V_1^t L_{11}^{tt} \sum_{k \neq t, i=1}^{r,2} A_{li}^{tk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_1^t L_{12}^{tt} \sum_{k \neq t, i=1}^{r,2} A_{2i}^{tk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ \boxed{V_2^t L_{21}^{tt} \sum_{k \neq t, i=1}^{r,2} A_{li}^{tk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_2^t L_{22}^{tt} \sum_{k \neq t, i=1}^{r,2} A_{2i}^{tk} \left[\sum_{u=s, j=1}^{t,2} B_{ij}^{ku} \left(\sum_{h=s, m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right]} \end{pmatrix}$$

分解思路：**t**国对**s**国出口中间品中包含的**t**国部门2的增加值所引致的国际总产出+**t**国对**r**国出口中间品中包含的**t**国部门2的增加值所引致的国际总产出

五：Xvi_GVC(即PLv_GVC分子)分解至国家_行业.目的地——简单与复杂

```
VLAmBBY分解至国家_行业.国家的代码： 即Xvd_GVC
VLAmBBY_forward<-array(0,dim=c(GN*G,1))
VL<-diag(VC)%*%L
Y_1<-array(rowSums(Y),dim=c(GN,GN))
BBY<-t(B%*%B%*%Y_1)
AmBBY<-Am*BBY
for (i in 1:G) {
  m=1+(i-1)*N ; n=N+(i-1)*N
  s=1+(i-1)*GN ; u=GN+(i-1)*GN
  AmBBY_1<-array(rowSums(AmBBY[,m:n]),dim=c(GN,GN))
  AmBBY_2<-t(AmBBY_1)
  AmBBY_F<-AmBBY_2
  for (j in 1:G) {
    mm=1+(j-1)*N ; nn=N+(j-1)*N
    AmBBY_F[mm:nn,mm:nn]<-0
  }
  AmBBY_D<-AmBBY_2-AmBBY_F
  VLAmBBY_1<-VLL*AmBBY_D
  VLAmBBY_2<-as.matrix(rowSums(VLAmBBY_1))
  VLAmBBY_forward[s:u,1]<-VLAmBBY_2[,1]
}
```

Xvi_GVC分解至国家_行业.目的地——VLAmBBY

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix}$$

$$\overline{\text{rowSums}(Y)} = \begin{pmatrix} Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \end{pmatrix}$$

$$A_m = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$\overline{\text{BrowSums}(Y)} =$$

$$\begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} \begin{pmatrix} Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \end{pmatrix}$$

$$\overline{\text{BrowSums}(Y)} =$$

$$\begin{pmatrix} \sum_{h=s,m=1}^{t,2} B_{1m}^{sh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{sh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{sh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{sh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{sh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{sh} Y_m^h \\ \sum_{h=s,m=1}^{t,2} B_{2m}^{sh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{sh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{sh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{sh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{sh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{sh} Y_m^h \\ \sum_{h=s,m=1}^{t,2} B_{1m}^{rh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{rh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{rh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{rh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{rh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{rh} Y_m^h \\ \sum_{h=s,m=1}^{t,2} B_{2m}^{rh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{rh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{rh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{rh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{rh} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{rh} Y_m^h \\ \sum_{h=s,m=1}^{t,2} B_{1m}^{th} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{th} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{th} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{th} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{th} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{1m}^{th} Y_m^h \\ \sum_{h=s,m=1}^{t,2} B_{2m}^{th} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{th} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{th} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{th} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{th} Y_m^h & \sum_{h=s,m=1}^{t,2} B_{2m}^{th} Y_m^h \end{pmatrix}$$

Xvi GVC分解至国家_行业.目的地——VLAmBBY

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$$\overline{(\text{BBrowSums}(\mathbf{Y}))}^T$$

$$\begin{pmatrix} 0 & 0 & A_{11}^{\text{sr}} & A_{12}^{\text{sr}} & A_{11}^{\text{st}} & A_{12}^{\text{st}} \\ 0 & 0 & A_{21}^{\text{sr}} & A_{22}^{\text{sr}} & A_{21}^{\text{st}} & A_{22}^{\text{st}} \\ A_{11}^{\text{rs}} & A_{12}^{\text{rs}} & 0 & 0 & A_{11}^{\text{rt}} & A_{12}^{\text{rt}} \\ A_{21}^{\text{rs}} & A_{22}^{\text{rs}} & 0 & 0 & A_{21}^{\text{rt}} & A_{22}^{\text{rt}} \\ A_{11}^{\text{ts}} & A_{12}^{\text{ts}} & A_{11}^{\text{tr}} & A_{12}^{\text{tr}} & 0 & 0 \\ A_{21}^{\text{ts}} & A_{22}^{\text{ts}} & A_{21}^{\text{tr}} & A_{22}^{\text{tr}} & 0 & 0 \end{pmatrix}$$

*

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$$\text{Am} * (\overline{\text{BBrowSums}(\text{Y})})^T = \begin{pmatrix} 0 & 0 & A_{11}^{sr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{sr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{11}^{st} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{st} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ 0 & 0 & A_{21}^{sr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{sr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{21}^{st} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{st} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{11}^{rs} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{rs} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & 0 & 0 & A_{11}^{rt} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{rt} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{21}^{rs} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{rs} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & 0 & 0 & A_{21}^{rt} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{rt} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{11}^{ts} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{ts} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{11}^{tr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{tr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & 0 & 0 \\ A_{21}^{ts} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{ts} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{21}^{tr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{tr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & 0 & 0 \end{pmatrix}$$

$$\text{rowSums}(\text{Am} * (\overline{\text{BBrowSums}(\text{Y})})^T [1:2]) = \left\{ \begin{array}{l} 0 \\ 0 \\ A_{11}^{rs} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) + A_{12}^{rs} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{21}^{rs} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) + A_{22}^{rs} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{11}^{ts} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) + A_{12}^{ts} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{21}^{ts} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) + A_{22}^{ts} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \end{array} \right\} = \left\{ \begin{array}{l} 0 \\ 0 \\ \sum_{i=1}^2 A_{1i}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \right] \\ \sum_{i=1}^2 A_{2i}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \right] \\ \sum_{i=1}^2 A_{1i}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \right] \\ \sum_{i=1}^2 A_{2i}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \right] \end{array} \right\}$$



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$$((\text{rowSums}(\text{Am} * (\text{BBrowSums}(\mathbf{Y}))^T[1:2]))^T)^F = \begin{pmatrix} 0 & 0 & \sum_{i=1}^2 A_{li}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{li}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ 0 & 0 & \sum_{i=1}^2 A_{li}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{li}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ 0 & 0 & 0 & 0 & \sum_{i=1}^2 A_{li}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ 0 & 0 & 0 & 0 & \sum_{i=1}^2 A_{li}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ 0 & 0 & \sum_{i=1}^2 A_{li}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & & \\ 0 & 0 & \sum_{i=1}^2 A_{li}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}
& \left(\begin{array}{cccc} \text{Sums}(\text{Am} * (\text{BBrowSums}(\text{Y}))^T[1:2]))^T - ((\text{rowSums}(\text{Am} * (\text{BBrowSums}(\text{Y}))^T[1:2]))^T)^F \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{i=1}^2 A_{li}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \\ 0 & 0 & \sum_{i=1}^2 A_{li}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{i=1}^2 A_{li}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ 0 & 0 & 0 & 0 & \sum_{i=1}^2 A_{li}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \end{pmatrix} \end{array} \right)^T
\end{aligned}$$

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$$\widehat{VL} * ((\text{rowSums}(\text{Am} * (\text{BBrowSums}(Y))^T [1:2]))^T)^D =$$

$$\begin{pmatrix} V_1^{sT} L_{11}^{ss} & V_1^{sT} L_{12}^{ss} & 0 & 0 & 0 & 0 \\ V_2^{sT} L_{21}^{ss} & V_2^{sT} L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^{rT} L_{11}^{rr} & V_1^{rT} L_{12}^{rr} & 0 & 0 \\ 0 & 0 & V_2^{rT} L_{21}^{rr} & V_2^{rT} L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^{tT} L_{11}^{tt} & V_1^{tT} L_{12}^{tt} \\ 0 & 0 & 0 & 0 & V_2^{tT} L_{21}^{tt} & V_2^{tT} L_{22}^{tt} \end{pmatrix} * \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{i=1}^2 A_{li}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \\ 0 & 0 & \sum_{i=1}^2 A_{li}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{i=1}^2 A_{li}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ 0 & 0 & 0 & 0 & \sum_{i=1}^2 A_{li}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \end{pmatrix}$$

$$\widehat{VL} * ((\text{rowSums}(\text{Am} * (\text{BBrowSums}(Y))^T [1:2]))^T)^D =$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^{rT} L_{11}^{rr} \sum_{i=1}^2 A_{li}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & V_1^{rT} L_{12}^{rr} \sum_{i=1}^2 A_{2i}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \\ 0 & 0 & V_2^{rT} L_{21}^{rr} \sum_{i=1}^2 A_{li}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & V_2^{rT} L_{22}^{rr} \sum_{i=1}^2 A_{2i}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^{tT} L_{11}^{tt} \sum_{i=1}^2 A_{li}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & V_1^{tT} L_{12}^{tt} \sum_{i=1}^2 A_{2i}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ 0 & 0 & 0 & 0 & V_2^{tT} L_{21}^{tt} \sum_{i=1}^2 A_{li}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & V_2^{tT} L_{22}^{tt} \sum_{i=1}^2 A_{2i}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \end{pmatrix}$$

Xvi_GVC分解至国家_行业.目的地——VLAmBBY

$$\text{rowSums}(\hat{\text{VL}} * ((\text{rowSums}(\text{Am} * (\text{BBrowSums}(\text{Y}))^T [1:2]))^T)^D) =$$

此矩阵的目的
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$$\begin{pmatrix} 0 \\ 0 \\ V_1^r L_{11}^{rr} \sum_{i=1}^2 A_{li}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_1^r L_{12}^{rr} \sum_{i=1}^2 A_{2i}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ V_2^r L_{21}^{rr} \sum_{i=1}^2 A_{li}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_2^r L_{22}^{rr} \sum_{i=1}^2 A_{2i}^{rs} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ V_1^t L_{11}^{tt} \sum_{i=1}^2 A_{li}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_1^t L_{12}^{tt} \sum_{i=1}^2 A_{2i}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ V_2^t L_{21}^{tt} \sum_{i=1}^2 A_{li}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_2^t L_{22}^{tt} \sum_{i=1}^2 A_{2i}^{ts} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{su} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \end{pmatrix}$$

t国向s国出口中间品中包含
的t国部门2的增加值的生产
活动中引致的国际总产出

Xvi_GVC分解至国家_行业.目的地——VLAmBBY

$$Am * (\overline{BBrowSums(Y)})^T = \begin{pmatrix} 0 & 0 & A_{11}^{sr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{sr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{11}^{st} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{st} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ 0 & 0 & A_{21}^{sr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{sr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{21}^{st} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{st} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{11}^{rs} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{rs} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & 0 & 0 & A_{11}^{rt} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{rt} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{21}^{rs} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{rs} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & 0 & 0 & A_{21}^{rt} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{rt} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{11}^{ts} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{ts} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{11}^{tr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{tr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & 0 & 0 \\ A_{21}^{ts} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{ts} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{21}^{tr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{tr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & 0 & 0 \end{pmatrix}$$

$$rowSums(Am * (\overline{BBrowSums(Y)})^T [3:4]) = \left\{ \begin{array}{l} A_{11}^{sr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) + A_{12}^{sr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{21}^{sr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) + A_{22}^{sr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ 0 \\ 0 \\ A_{11}^{tr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) + A_{12}^{tr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{21}^{tr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) + A_{22}^{tr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \end{array} \right\} = \left\{ \begin{array}{l} \sum_{i=1}^2 A_{1i}^{sr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \right] \\ \sum_{i=1}^2 A_{2i}^{sr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \right] \\ 0 \\ 0 \\ \sum_{i=1}^2 A_{1i}^{tr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \right] \\ \sum_{i=1}^2 A_{2i}^{tr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \right] \end{array} \right\}$$



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[illegible]

$$\begin{aligned}
& ((\text{rowSums}(\text{Am} * (\text{BBrowSums}(\text{Y}))^\text{T}[3:4]))^\text{T})^\text{D} = (\text{rowSums}(\text{Am} * (\text{BBrowSums}(\text{Y}))^\text{T}[3:4]))^\text{T} - ((\text{rowSums}(\text{Am} * (\text{BBrowSums}(\text{Y}))^\text{T}[3:4]))^\text{T})^\text{F} \\
& = \begin{pmatrix} \sum_{i=1}^2 \text{A}_{li}^{\text{sr}} \left[\sum_{u=s,j=1}^{t,2} \text{B}_{ij}^{\text{ru}} \left(\sum_{h=s,m=1}^{t,2} \text{B}_{jm}^{\text{uh}} \text{Y}_m^h \right) \right] & \sum_{i=1}^2 \text{A}_{2i}^{\text{sr}} \left[\sum_{u=s,j=1}^{t,2} \text{B}_{ij}^{\text{ru}} \left(\sum_{h=s,m=1}^{t,2} \text{B}_{jm}^{\text{uh}} \text{Y}_m^h \right) \right] & 0 & 0 & 0 & 0 \\ \sum_{i=1}^2 \text{A}_{li}^{\text{sr}} \left[\sum_{u=s,j=1}^{t,2} \text{B}_{ij}^{\text{ru}} \left(\sum_{h=s,m=1}^{t,2} \text{B}_{jm}^{\text{uh}} \text{Y}_m^h \right) \right] & \sum_{i=1}^2 \text{A}_{2i}^{\text{sr}} \left[\sum_{u=s,j=1}^{t,2} \text{B}_{ij}^{\text{ru}} \left(\sum_{h=s,m=1}^{t,2} \text{B}_{jm}^{\text{uh}} \text{Y}_m^h \right) \right] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{i=1}^2 \text{A}_{li}^{\text{tr}} \left[\sum_{u=s,j=1}^{t,2} \text{B}_{ij}^{\text{ru}} \left(\sum_{h=s,m=1}^{t,2} \text{B}_{jm}^{\text{uh}} \text{Y}_m^h \right) \right] & \sum_{i=1}^2 \text{A}_{2i}^{\text{tr}} \left[\sum_{u=s,j=1}^{t,2} \text{B}_{ij}^{\text{ru}} \left(\sum_{h=s,m=1}^{t,2} \text{B}_{jm}^{\text{uh}} \text{Y}_m^h \right) \right] \\ 0 & 0 & 0 & 0 & \sum_{i=1}^2 \text{A}_{li}^{\text{tr}} \left[\sum_{u=s,j=1}^{t,2} \text{B}_{ij}^{\text{ru}} \left(\sum_{h=s,m=1}^{t,2} \text{B}_{jm}^{\text{uh}} \text{Y}_m^h \right) \right] & \sum_{i=1}^2 \text{A}_{2i}^{\text{tr}} \left[\sum_{u=s,j=1}^{t,2} \text{B}_{ij}^{\text{ru}} \left(\sum_{h=s,m=1}^{t,2} \text{B}_{jm}^{\text{uh}} \text{Y}_m^h \right) \right] \end{pmatrix}
\end{aligned}$$

Xvi_GVC分解至国家_行业.目的地——VLAmBBY

$$\widehat{VL} * ((\text{rowSums}(\text{Am} * (\text{BBrowSums}(Y))^T [3:4]))^T)^D =$$

$$\begin{pmatrix} V_1^s L_{11}^{ss} & V_1^s L_{12}^{ss} & 0 & 0 & 0 & 0 \\ V_2^s L_{21}^{ss} & V_2^s L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r L_{11}^{rr} & V_1^r L_{12}^{rr} & 0 & 0 \\ 0 & 0 & V_2^r L_{21}^{rr} & V_2^r L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t L_{11}^{tt} & V_1^t L_{12}^{tt} \\ 0 & 0 & 0 & 0 & V_2^t L_{21}^{tt} & V_2^t L_{22}^{tt} \end{pmatrix} * \begin{pmatrix} \sum_{i=1}^2 A_{li}^{sr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{sr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 & 0 & 0 \\ \sum_{i=1}^2 A_{li}^{sr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{sr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{i=1}^2 A_{li}^{tr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{tr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \\ 0 & 0 & \sum_{i=1}^2 A_{li}^{tr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{tr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \end{pmatrix}$$

$$\widehat{VL} * ((\text{rowSums}(\text{Am} * (\text{BBrowSums}(Y))^T [3:4]))^T)^D =$$

$$\begin{pmatrix} V_1^s L_{11}^{ss} \sum_{i=1}^2 A_{li}^{sr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & V_1^s L_{12}^{ss} \sum_{i=1}^2 A_{2i}^{sr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 & 0 & 0 \\ V_2^s L_{21}^{ss} \sum_{i=1}^2 A_{li}^{sr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & V_2^s L_{22}^{ss} \sum_{i=1}^2 A_{2i}^{sr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^t L_{11}^{tt} \sum_{i=1}^2 A_{li}^{tr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & V_1^t L_{12}^{tt} \sum_{i=1}^2 A_{2i}^{tr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \\ 0 & 0 & V_2^t L_{21}^{tt} \sum_{i=1}^2 A_{li}^{tr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & V_2^t L_{22}^{tt} \sum_{i=1}^2 A_{2i}^{tr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \end{pmatrix}$$

Xvi_GVC分解至国家_行业.目的地——VLAmBBY

$$\text{rowSums}(\widehat{\text{VL}} * ((\text{rowSums}(\text{Am} * (\text{BBrowSums}(\text{Y}))^T [3:4]))^T)^D) =$$

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$$\begin{pmatrix} V_1^s L_{11}^{ss} \sum_{i=1}^2 A_{li}^{sr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_1^s L_{12}^{ss} \sum_{i=1}^2 A_{2i}^{sr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ V_2^s L_{21}^{ss} \sum_{i=1}^2 A_{li}^{sr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_2^s L_{22}^{ss} \sum_{i=1}^2 A_{2i}^{sr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ 0 \\ 0 \\ V_1^t L_{11}^{tt} \sum_{i=1}^2 A_{li}^{tr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_1^t L_{12}^{tt} \sum_{i=1}^2 A_{2i}^{tr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ V_2^t L_{21}^{tt} \sum_{i=1}^2 A_{li}^{tr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_2^t L_{22}^{tt} \sum_{i=1}^2 A_{2i}^{tr} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{ru} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \end{pmatrix}$$

t国向r国出口中间品中包含
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Xvi_GVC分解至国家_行业.目的地——VLAmBBY

$$Am * (\overline{BBrowSums(Y)})^T = \begin{pmatrix} 0 & 0 & A_{11}^{sr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{sr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{11}^{st} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{st} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ 0 & 0 & A_{21}^{sr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{sr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{21}^{st} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{st} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{11}^{rs} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{rs} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & 0 & 0 & A_{11}^{rt} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{rt} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{21}^{rs} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{rs} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & 0 & 0 & A_{21}^{rt} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{rt} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{11}^{ts} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{ts} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{11}^{tr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{12}^{tr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & 0 & 0 \\ A_{21}^{ts} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{ts} \sum_{u=s,j=1}^{t,2} B_{2j}^{su} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{21}^{tr} \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & A_{22}^{tr} \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) & 0 & 0 \end{pmatrix}$$

$$rowSums(Am * (\overline{BBrowSums(Y)})^T[, 5:6]) = \begin{pmatrix} A_{11}^{st} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) + A_{12}^{st} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{21}^{st} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) + A_{22}^{st} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{11}^{rt} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) + A_{12}^{rt} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ A_{21}^{rt} \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) + A_{22}^{rt} \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^2 A_{1i}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \right] \\ \sum_{i=1}^2 A_{2i}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \right] \\ \sum_{i=1}^2 A_{1i}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \right] \\ \sum_{i=1}^2 A_{2i}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} (\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h) \right] \\ 0 \\ 0 \end{pmatrix}$$



[illegible]

$$((\text{rowSums}(\text{Am} * (\text{BBrowSums}(\text{Y}))^T[, 5:6]))^T)^D = (\text{rowSums}(\text{Am} * (\text{BBrowSums}(\text{Y}))^T[, 5:6]))^T - ((\text{rowSums}(\text{Am} * (\text{BBrowSums}(\text{Y}))^T[, 5:6]))^T)^F$$

$$= \begin{pmatrix} \sum_{i=1}^2 A_{li}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 & 0 & 0 \\ \sum_{i=1}^2 A_{li}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{i=1}^2 A_{li}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \\ 0 & 0 & \sum_{i=1}^2 A_{li}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Xvi_GVC分解至国家_行业.目的地——VLAmBBY

$$\widehat{VL} * ((\text{rowSums}(\text{Am} * (\text{BBrowSums}(Y))^T [, 5 : 6]))^T)^D =$$

$$\begin{pmatrix} V_1^s L_{11}^{ss} & V_1^s L_{12}^{ss} & 0 & 0 & 0 & 0 \\ V_2^s L_{21}^{ss} & V_2^s L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r L_{11}^{rr} & V_1^r L_{12}^{rr} & 0 & 0 \\ 0 & 0 & V_2^r L_{21}^{rr} & V_2^r L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t L_{11}^{tt} & V_1^t L_{12}^{tt} \\ 0 & 0 & 0 & 0 & V_2^t L_{21}^{tt} & V_2^t L_{22}^{tt} \end{pmatrix} * \begin{pmatrix} \sum_{i=1}^2 A_{li}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 & 0 & 0 \\ \sum_{i=1}^2 A_{li}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{i=1}^2 A_{li}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \\ 0 & 0 & \sum_{i=1}^2 A_{li}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & \sum_{i=1}^2 A_{2i}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\widehat{VL} * ((\text{rowSums}(\text{Am} * (\text{BBrowSums}(Y))^T [, 5 : 6]))^T)^D =$$

$$\begin{pmatrix} V_1^s L_{11}^{ss} \sum_{i=1}^2 A_{li}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & V_1^s L_{12}^{ss} \sum_{i=1}^2 A_{2i}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 & 0 & 0 \\ V_2^s L_{21}^{ss} \sum_{i=1}^2 A_{li}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & V_2^s L_{22}^{ss} \sum_{i=1}^2 A_{2i}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r L_{11}^{rr} \sum_{i=1}^2 A_{li}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & V_1^r L_{12}^{rr} \sum_{i=1}^2 A_{2i}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \\ 0 & 0 & V_2^r L_{21}^{rr} \sum_{i=1}^2 A_{li}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & V_2^r L_{22}^{rr} \sum_{i=1}^2 A_{2i}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Xvi_GVC分解至国家_行业.目的地——VLAmBBY

$$\text{rowSums}(\widehat{\text{VL}} * ((\text{rowSums}(\text{Am} * (\text{BBrowSums}(\text{Y}))^T [5:6]))^T)^D) =$$

此矩阵的目的
地均为t国

$$\begin{pmatrix} V_1^s L_{11}^{ss} \sum_{i=1}^2 A_{li}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_1^s L_{12}^{ss} \sum_{i=1}^2 A_{2i}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ V_2^s L_{21}^{ss} \sum_{i=1}^2 A_{li}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_2^s L_{22}^{ss} \sum_{i=1}^2 A_{2i}^{st} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ V_1^r L_{11}^{rr} \sum_{i=1}^2 A_{li}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_1^r L_{12}^{rr} \sum_{i=1}^2 A_{2i}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ V_2^r L_{21}^{rr} \sum_{i=1}^2 A_{li}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] + V_2^r L_{22}^{rr} \sum_{i=1}^2 A_{2i}^{rt} \left[\sum_{u=s,j=1}^{t,2} B_{ij}^{tu} \left(\sum_{h=s,m=1}^{t,2} B_{jm}^{uh} Y_m^h \right) \right] \\ 0 \\ 0 \end{pmatrix}$$

r国向t国出口中间品中包含
的r国部门2的增加值的生产
活动中引致的国际总产出

六：VA_GVC(即PLv_GVC分母)分解至国家_行业.国家——对V(B-L)Y分解

V(B-L)Y分解至国家_行业.国家的代码：

```
VLAmbY_forward<-array(0,dim=c(GN*G,1))
```

```
VL<-diag(VC)%*%L
```

```
Y_1<-array(rowSums(Y),dim=c(GN,GN))
```

```
BY<-t(B%*%Y_1)
```

```
AmBY<-Am*BY
```

```
for (i in 1:G) {
```

```
  m=1+(i-1)*N ; n=N+(i-1)*N
```

```
  s=1+(i-1)*GN ; u=GN+(i-1)*GN
```

```
  AmBY_1<-array(rowSums(AmBY[,m:n]),dim=c(GN,GN))
```

```
  AmBY_2<-t(AmBY_1)
```

```
  AmBY_F<-AmBY_2
```

```
  for (j in 1:G) {
```

```
    mm=1+(j-1)*N ; nn=N+(j-1)*N
```

```
    AmBY_F[mm:nn,mm:nn]<-0
```

```
  }
```

```
  AmBY_D<-AmBY_2-AmBY_F
```

```
  VLAmbY_1<-VL*AmBY_D
```

```
  VLAmbY_2<-as.matrix(rowSums(VLAmbY_1))
```

```
  VLAmbY_forward[s:u,1]<-VLAmbY_2[,1]
```

```
}
```

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\text{rowSums}(\widehat{V}(\widehat{B-L})\widehat{\text{rowSums}}(Y)) = \text{rowSums}(\widehat{V}L\widehat{A}mBY)$$

$$= \begin{pmatrix} V_1^s L_{11}^{ss} \sum_{u \neq s, j=1}^{t,2} A_{1j}^{su} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) + V_1^s L_{12}^{ss} \sum_{u \neq s, j=1}^{t,2} A_{2j}^{su} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) \\ V_2^s L_{21}^{ss} \sum_{u \neq s, j=1}^{t,2} A_{1j}^{su} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) + V_2^s L_{22}^{ss} \sum_{u \neq s, j=1}^{t,2} A_{2j}^{su} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) \\ V_1^r L_{11}^{rr} \sum_{u \neq r, j=1}^{t,2} A_{1j}^{ru} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) + V_1^r L_{12}^{rr} \sum_{u \neq r, j=1}^{t,2} A_{2j}^{ru} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) \\ V_2^r L_{21}^{rr} \sum_{u \neq r, j=1}^{t,2} A_{1j}^{ru} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) + V_2^r L_{22}^{rr} \sum_{u \neq r, j=1}^{t,2} A_{2j}^{ru} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) \\ V_1^t L_{11}^{tt} \sum_{u \neq t, j=1}^{r,2} A_{1j}^{tu} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) + V_1^t L_{12}^{tt} \sum_{u \neq t, j=1}^{r,2} A_{2j}^{tu} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) \\ V_2^t L_{21}^{tt} \sum_{u \neq t, j=1}^{r,2} A_{1j}^{tu} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) + V_2^t L_{22}^{tt} \sum_{u \neq t, j=1}^{r,2} A_{2j}^{tu} \left(\sum_{k=s, i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) \end{pmatrix}$$

分解思路:

s国向r国出口中间品并参与GVC生产活动中包含的s国部门2的增加值+s国向t国出口中间品并参与GVC生产活动中包含的s国部门2的增加值

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix}$$

$$\overline{\text{rowSums}(Y)} = \begin{pmatrix} Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \end{pmatrix}$$

$$Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$\overline{\text{BrowSums}(Y)} =$$

$$\begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} \begin{pmatrix} Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \end{pmatrix}$$

$$\overline{\text{BrowSums}(Y)} =$$

$$\begin{pmatrix} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k \\ \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k \\ \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k \\ \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k \\ \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k \\ \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k & \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \end{pmatrix}$$

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\begin{aligned} & \overline{\text{(BrowSums}(Y))}^T = \left(\sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k \quad \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k \quad \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k \quad \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k \quad \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k \quad \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \right) \\ & \text{Am} * \overline{\text{(BrowSums}(Y))}^T = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix} * \left(\sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k \quad \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k \quad \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k \quad \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k \quad \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k \quad \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \right) \end{aligned}$$

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$Am * (\overline{BrowSums(Y)})^T = \begin{pmatrix} 0 & 0 & A_{11}^{sr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & A_{12}^{sr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & A_{11}^{st} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & A_{12}^{st} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ 0 & 0 & A_{21}^{sr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & A_{22}^{sr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & A_{21}^{st} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & A_{22}^{st} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ A_{11}^{rs} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & A_{12}^{rs} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & 0 & 0 & A_{11}^{rt} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & A_{12}^{rt} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ A_{21}^{rs} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & A_{22}^{rs} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & 0 & 0 & A_{21}^{rt} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & A_{22}^{rt} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ A_{11}^{ts} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & A_{12}^{ts} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & A_{11}^{tr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & A_{12}^{tr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & 0 & 0 \\ A_{21}^{ts} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & A_{22}^{ts} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & A_{21}^{tr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & A_{22}^{tr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & 0 & 0 \end{pmatrix}$$

$$rowSums(Am * (\overline{BrowSums(Y)})^T [1:2]) = \begin{pmatrix} 0 \\ 0 \\ A_{11}^{rs} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k + A_{12}^{rs} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k \\ A_{21}^{rs} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k + A_{22}^{rs} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k \\ A_{11}^{ts} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k + A_{12}^{ts} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k \\ A_{21}^{ts} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k + A_{22}^{ts} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sum_{j=1}^2 A_{1j}^{rs} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) \\ \sum_{j=1}^2 A_{2j}^{rs} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) \\ \sum_{j=1}^2 A_{1j}^{ts} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) \\ \sum_{j=1}^2 A_{2j}^{ts} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) \end{pmatrix}$$

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

[illegible]

$$\overline{(\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[1:2]))^T} = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k) \end{pmatrix}$$

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[1:2]))^T)^F} = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & 0 & 0 \end{pmatrix}$$

$$\overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[1:2]))^T)^D} = \overline{(\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[1:2]))^T} - \overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[1:2]))^T)^F}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) \end{pmatrix}$$

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\widehat{VL} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [1:2]))^T)^D$$

$$= \begin{pmatrix} V_1^s L_{11}^{ss} & V_1^s L_{12}^{ss} & 0 & 0 & 0 & 0 \\ V_2^s L_{21}^{ss} & V_2^s L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r L_{11}^{rr} & V_1^r L_{12}^{rr} & 0 & 0 \\ 0 & 0 & V_2^r L_{21}^{rr} & V_2^r L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t L_{11}^{tt} & V_1^t L_{12}^{tt} \\ 0 & 0 & 0 & 0 & V_2^t L_{21}^{tt} & V_2^t L_{22}^{tt} \end{pmatrix} * \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{u \neq r, j=1}^{t,2} A_{lj}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & \sum_{u \neq r, j=1}^{t,2} A_{2j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & 0 & 0 \\ 0 & 0 & \sum_{u \neq r, j=1}^{t,2} A_{lj}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & \sum_{u \neq r, j=1}^{t,2} A_{2j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{u \neq t, j=1}^{r,2} A_{lj}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) & \sum_{u \neq t, j=1}^{r,2} A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) \\ 0 & 0 & 0 & 0 & \sum_{u \neq t, j=1}^{r,2} A_{lj}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) & \sum_{u \neq t, j=1}^{r,2} A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) \end{pmatrix}$$

$$\widehat{VL} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [1:2]))^T)^D = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r L_{11}^{rr} \sum_{j=1}^2 A_{lj}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & V_1^r L_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & 0 & 0 \\ 0 & 0 & V_2^r L_{21}^{rr} \sum_{j=1}^2 A_{lj}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & V_2^r L_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t L_{11}^{tt} \sum_{j=1}^2 A_{lj}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) & V_1^t L_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) \\ 0 & 0 & 0 & 0 & V_2^t L_{21}^{tt} \sum_{j=1}^2 A_{lj}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) & V_2^t L_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) \end{pmatrix}$$

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\text{rowSums}(\widehat{\text{VL}} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [1:2]))^T)^D)$$

此矩阵的目的
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$$= \begin{pmatrix} 0 & 0 \\ V_1^r L_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) + V_1^r L_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) \\ V_2^r L_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) + V_2^r L_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) \\ V_1^t L_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) + V_1^t L_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) \\ V_2^t L_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) + V_2^t L_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ V_1^r L_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} X_j^s + V_1^r L_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} X_j^s \\ V_2^r L_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} X_j^s + V_2^r L_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} X_j^s \\ V_1^t L_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} X_j^s + V_1^t L_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} X_j^s \\ V_2^t L_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} X_j^s + V_2^t L_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} X_j^s \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ V_1^r L_{11}^{rr} \sum_{j=1}^2 Z_{1j}^{rs} + V_1^r L_{12}^{rr} \sum_{j=1}^2 Z_{2j}^{rs} \\ V_2^r L_{21}^{rr} \sum_{j=1}^2 Z_{1j}^{rs} + V_2^r L_{22}^{rr} \sum_{j=1}^2 Z_{2j}^{rs} \\ V_1^t L_{11}^{tt} \sum_{j=1}^2 Z_{1j}^{ts} + V_1^t L_{12}^{tt} \sum_{j=1}^2 Z_{2j}^{ts} \\ V_2^t L_{21}^{tt} \sum_{j=1}^2 Z_{1j}^{ts} + V_2^t L_{22}^{tt} \sum_{j=1}^2 Z_{2j}^{ts} \end{pmatrix}$$

r国向s国出口中间品并
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含的r国部门1的增加值

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$Am * (\overline{BrowSums(Y)})^T = \begin{pmatrix} 0 & 0 & A_{11}^{sr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & A_{12}^{sr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & A_{11}^{st} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & A_{12}^{st} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ 0 & 0 & A_{21}^{sr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & A_{22}^{sr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & A_{21}^{st} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & A_{22}^{st} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ A_{11}^{rs} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & A_{12}^{rs} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & 0 & 0 & A_{11}^{rt} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & A_{12}^{rt} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ A_{21}^{rs} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & A_{22}^{rs} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & 0 & 0 & A_{21}^{rt} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & A_{22}^{rt} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ A_{11}^{ts} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & A_{12}^{ts} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & A_{11}^{tr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & A_{12}^{tr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & 0 & 0 \\ A_{21}^{ts} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & A_{22}^{ts} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & A_{21}^{tr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & A_{22}^{tr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & 0 & 0 \end{pmatrix}$$

$$rowSums(Am * (\overline{BrowSums(Y)})^T [3:4]) = \begin{pmatrix} A_{11}^{sr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k + A_{12}^{sr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k \\ A_{21}^{sr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k + A_{22}^{sr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k \\ 0 \\ 0 \\ A_{11}^{tr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k + A_{12}^{tr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k \\ A_{21}^{tr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k + A_{22}^{tr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) \\ \sum_{j=1}^2 A_{2j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) \\ 0 \\ 0 \\ \sum_{j=1}^2 A_{1j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) \\ \sum_{j=1}^2 A_{2j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) \end{pmatrix}$$

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

[illegible]

$$\overline{(\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^\top[3 : 4]))^\top = \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) \end{pmatrix}$$

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$((\text{rowSums}(\text{Am} * (\text{BrowSums}(\mathbf{Y}))^T[3:4]))^T)^F = \begin{pmatrix} 0 & 0 & 0 & 0 & \sum_{j=1}^2 \mathbf{A}_{1j}^{\text{tr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) & \sum_{j=1}^2 \mathbf{A}_{2j}^{\text{tr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 \mathbf{A}_{1j}^{\text{tr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) & \sum_{j=1}^2 \mathbf{A}_{2j}^{\text{tr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) \\ \sum_{j=1}^2 \mathbf{A}_{1j}^{\text{sr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) & \sum_{j=1}^2 \mathbf{A}_{2j}^{\text{sr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) & 0 & 0 & \sum_{j=1}^2 \mathbf{A}_{1j}^{\text{tr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) & \sum_{j=1}^2 \mathbf{A}_{2j}^{\text{tr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) \\ \sum_{j=1}^2 \mathbf{A}_{1j}^{\text{sr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) & \sum_{j=1}^2 \mathbf{A}_{2j}^{\text{sr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) & 0 & 0 & \sum_{j=1}^2 \mathbf{A}_{1j}^{\text{tr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) & \sum_{j=1}^2 \mathbf{A}_{2j}^{\text{tr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) \\ \sum_{j=1}^2 \mathbf{A}_{1j}^{\text{sr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) & \sum_{j=1}^2 \mathbf{A}_{2j}^{\text{sr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 \mathbf{A}_{1j}^{\text{sr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) & \sum_{j=1}^2 \mathbf{A}_{2j}^{\text{sr}} (\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{\text{rk}} \mathbf{Y}_i^k) & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[, 3:4]))^T)^D = (\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[, 3:4]))^T - ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[, 3:4]))^T)^F$$

$$= \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{\text{sr}} \sum_{k=s,i=1}^{t,2} B_{ji}^{\text{rk}} Y_i^k & \sum_{j=1}^2 A^{\text{sr}} \sum_{k=s,i=1}^{t,2} B^{\text{rk}} Y_i^k & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{\text{sr}} (\sum_{k=s,i=1}^{t,2} B_{ji}^{\text{rk}} Y_i^k) & \sum_{j=1}^2 A_{2j}^{\text{sr}} (\sum_{k=s,i=1}^{t,2} B_{ji}^{\text{rk}} Y_i^k) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{\text{tr}} (\sum_{k=s,i=1}^{t,2} B_{ji}^{\text{rk}} Y_i^k) & \sum_{j=1}^2 A_{2j}^{\text{tr}} (\sum_{k=s,i=1}^{t,2} B_{ji}^{\text{rk}} Y_i^k) \\ 0 & 0 & 0 & 0 & \sum_{j=1}^2 A_{1j}^{\text{tr}} (\sum_{k=s,i=1}^{t,2} B_{ji}^{\text{rk}} Y_i^k) & \sum_{j=1}^2 A_{2j}^{\text{tr}} (\sum_{k=s,i=1}^{t,2} B_{ji}^{\text{rk}} Y_i^k) \end{pmatrix}$$

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\widehat{VL} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(Y))^T[3:4]))^T)^D$$

$$= \begin{pmatrix} V_1^s L_{11}^{ss} & V_1^s L_{12}^{ss} & 0 & 0 & 0 & 0 \\ V_2^s L_{21}^{ss} & V_2^s L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r L_{11}^{rr} & V_1^r L_{12}^{rr} & 0 & 0 \\ 0 & 0 & V_2^r L_{21}^{rr} & V_2^r L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t L_{11}^{tt} & V_1^t L_{12}^{tt} \\ 0 & 0 & 0 & 0 & V_2^t L_{21}^{tt} & V_2^t L_{22}^{tt} \end{pmatrix} * \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & 0 & 0 \end{pmatrix}$$

$$\widehat{VL} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(Y))^T[3:4]))^T)^D = \begin{pmatrix} V_1^s L_{11}^{ss} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & V_1^s L_{12}^{ss} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & 0 & 0 & 0 & 0 \\ V_2^s L_{21}^{ss} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & V_2^s L_{22}^{ss} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k) & 0 & 0 \end{pmatrix}$$

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

rowSums($\widehat{VL} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [, 3:4]))^T)^D$)

→ 此矩阵的目的
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$$= \begin{pmatrix} V_1^s L_{11}^{ss} \sum_{j=1}^2 A_{1j}^{sr} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k \right) + V_1^s L_{12}^{ss} \sum_{j=1}^2 A_{2j}^{sr} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k \right) \\ V_2^s L_{21}^{ss} \sum_{j=1}^2 A_{1j}^{sr} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k \right) + V_2^s L_{22}^{ss} \sum_{j=1}^2 A_{2j}^{sr} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k \right) \\ 0 \\ 0 \\ V_1^t L_{11}^{tt} \sum_{j=1}^2 A_{1j}^{tr} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k \right) + V_1^t L_{12}^{tt} \sum_{j=1}^2 A_{2j}^{tr} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k \right) \\ V_2^t L_{21}^{tt} \sum_{j=1}^2 A_{1j}^{tr} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k \right) + V_2^t L_{22}^{tt} \sum_{j=1}^2 A_{2j}^{tr} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k \right) \end{pmatrix} = \begin{pmatrix} V_1^s L_{11}^{ss} \sum_{j=1}^2 A_{1j}^{sr} X_j^r + V_1^s L_{12}^{ss} \sum_{j=1}^2 A_{2j}^{sr} X_j^r \\ V_2^s L_{21}^{ss} \sum_{j=1}^2 A_{1j}^{sr} X_j^r + V_2^s L_{22}^{ss} \sum_{j=1}^2 A_{2j}^{sr} X_j^r \\ 0 \\ 0 \\ V_1^t L_{11}^{tt} \sum_{j=1}^2 A_{1j}^{tr} X_j^r + V_1^t L_{12}^{tt} \sum_{j=1}^2 A_{2j}^{tr} X_j^r \\ V_2^t L_{21}^{tt} \sum_{j=1}^2 A_{1j}^{tr} X_j^r + V_2^t L_{22}^{tt} \sum_{j=1}^2 A_{2j}^{tr} X_j^r \end{pmatrix} = \begin{pmatrix} V_1^s L_{11}^{ss} \sum_{j=1}^2 Z_{1j}^{sr} + V_1^s L_{12}^{ss} \sum_{j=1}^2 Z_{2j}^{sr} \\ V_2^s L_{21}^{ss} \sum_{j=1}^2 Z_{1j}^{sr} + V_2^s L_{22}^{ss} \sum_{j=1}^2 Z_{2j}^{sr} \\ 0 \\ 0 \\ V_1^t L_{11}^{tt} \sum_{j=1}^2 Z_{1j}^{tr} + V_1^t L_{12}^{tt} \sum_{j=1}^2 Z_{2j}^{tr} \\ V_2^t L_{21}^{tt} \sum_{j=1}^2 Z_{1j}^{tr} + V_2^t L_{22}^{tt} \sum_{j=1}^2 Z_{2j}^{tr} \end{pmatrix}$$

→ s国向r国出口中间品并
参与GVC生产活动中包
含的s国部门2的增加值

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$Am * (\overline{BrowSums(Y)})^T = \begin{pmatrix} 0 & 0 & A_{11}^{sr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & A_{12}^{sr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & A_{11}^{st} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & A_{12}^{st} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ 0 & 0 & A_{21}^{sr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & A_{22}^{sr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & A_{21}^{st} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & A_{22}^{st} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ A_{11}^{rs} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & A_{12}^{rs} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & 0 & 0 & A_{11}^{rt} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & A_{12}^{rt} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ A_{21}^{rs} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & A_{22}^{rs} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & 0 & 0 & A_{21}^{rt} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k & A_{22}^{rt} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ A_{11}^{ts} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & A_{12}^{ts} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & A_{11}^{tr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & A_{12}^{tr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & 0 & 0 \\ A_{21}^{ts} \sum_{k=s,i=1}^{t,2} B_{li}^{sk} Y_i^k & A_{22}^{ts} \sum_{k=s,i=1}^{t,2} B_{2i}^{sk} Y_i^k & A_{21}^{tr} \sum_{k=s,i=1}^{t,2} B_{li}^{rk} Y_i^k & A_{22}^{tr} \sum_{k=s,i=1}^{t,2} B_{2i}^{rk} Y_i^k & 0 & 0 \end{pmatrix}$$

$$rowSums(Am * (\overline{BrowSums(Y)})^T [5:6]) = \begin{pmatrix} A_{11}^{st} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k + A_{12}^{st} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ A_{21}^{st} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k + A_{22}^{st} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ A_{11}^{rt} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k + A_{12}^{rt} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ A_{21}^{rt} \sum_{k=s,i=1}^{t,2} B_{li}^{tk} Y_i^k + A_{22}^{rt} \sum_{k=s,i=1}^{t,2} B_{2i}^{tk} Y_i^k \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) \\ \sum_{j=1}^2 A_{2j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) \\ \sum_{j=1}^2 A_{1j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) \\ \sum_{j=1}^2 A_{2j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) \\ 0 \\ 0 \end{pmatrix}$$

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[5:6]))^T)^F} = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 \end{pmatrix}$$

$$\overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[5:6]))^T)^D} = \overline{(\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[5:6]))^T} - \overline{((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T[5:6]))^T)^F}$$

$$= \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{st} \sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k & \sum_{j=1}^2 A_{2j}^{st} \sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\widehat{VL} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [5:6]))^T)^D$$

$$= \begin{pmatrix} V_1^s L_{11}^{ss} & V_1^s L_{12}^{ss} & 0 & 0 & 0 & 0 \\ V_2^s L_{21}^{ss} & V_2^s L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r L_{11}^{rr} & V_1^r L_{12}^{rr} & 0 & 0 \\ 0 & 0 & V_2^r L_{21}^{rr} & V_2^r L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t L_{11}^{tt} & V_1^t L_{12}^{tt} \\ 0 & 0 & 0 & 0 & V_2^t L_{21}^{tt} & V_2^t L_{22}^{tt} \end{pmatrix} * \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\widehat{VL} * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(\text{Y}))^T [5:6]))^T)^D$$

$$= \begin{pmatrix} V_1^s L_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & V_1^s L_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 & 0 & 0 \\ V_2^s L_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & V_2^s L_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r L_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & V_1^r L_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 \\ 0 & 0 & V_2^r L_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & V_2^r L_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rt} (\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\text{rowSums}(\widehat{\mathbf{V}}\mathbf{L} * ((\text{rowSums}(\mathbf{A}\mathbf{m} * (\text{BrowSums}(\mathbf{Y}))^T [5:6]))^T)^D) =$$

$$\begin{pmatrix} \mathbf{V}_1^s \mathbf{L}_{11}^{ss} \sum_{j=1}^2 \mathbf{A}_{1j}^{st} \left(\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{tk} \mathbf{Y}_i^k \right) + \mathbf{V}_1^s \mathbf{L}_{12}^{ss} \sum_{j=1}^2 \mathbf{A}_{2j}^{st} \left(\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{tk} \mathbf{Y}_i^k \right) \\ \mathbf{V}_2^s \mathbf{L}_{21}^{ss} \sum_{j=1}^2 \mathbf{A}_{1j}^{st} \left(\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{tk} \mathbf{Y}_i^k \right) + \mathbf{V}_2^s \mathbf{L}_{22}^{ss} \sum_{j=1}^2 \mathbf{A}_{2j}^{st} \left(\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{tk} \mathbf{Y}_i^k \right) \\ \mathbf{V}_1^r \mathbf{L}_{11}^{rr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rt} \left(\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{tk} \mathbf{Y}_i^k \right) + \mathbf{V}_1^r \mathbf{L}_{12}^{rr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rt} \left(\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{tk} \mathbf{Y}_i^k \right) \\ \mathbf{V}_2^r \mathbf{L}_{21}^{rr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rt} \left(\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{tk} \mathbf{Y}_i^k \right) + \mathbf{V}_2^r \mathbf{L}_{22}^{rr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rt} \left(\sum_{k=s,i=1}^{t,2} \mathbf{B}_{ji}^{tk} \mathbf{Y}_i^k \right) \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \mathbf{V}_1^s \mathbf{L}_{11}^{ss} \sum_{j=1}^2 \mathbf{A}_{1j}^{st} \mathbf{X}_j^t + \mathbf{V}_1^s \mathbf{L}_{12}^{ss} \sum_{j=1}^2 \mathbf{A}_{2j}^{st} \mathbf{X}_j^t \\ \mathbf{V}_2^s \mathbf{L}_{21}^{ss} \sum_{j=1}^2 \mathbf{A}_{1j}^{st} \mathbf{X}_j^t + \mathbf{V}_2^s \mathbf{L}_{22}^{ss} \sum_{j=1}^2 \mathbf{A}_{2j}^{st} \mathbf{X}_j^t \\ \mathbf{V}_1^r \mathbf{L}_{11}^{rr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rt} \mathbf{X}_j^t + \mathbf{V}_1^r \mathbf{L}_{12}^{rr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rt} \mathbf{X}_j^t \\ \mathbf{V}_2^r \mathbf{L}_{21}^{rr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rt} \mathbf{X}_j^t + \mathbf{V}_2^r \mathbf{L}_{22}^{rr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rt} \mathbf{X}_j^t \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \mathbf{V}_1^s \mathbf{L}_{11}^{ss} \sum_{j=1}^2 \mathbf{Z}_{1j}^{st} + \mathbf{V}_1^s \mathbf{L}_{12}^{ss} \sum_{j=1}^2 \mathbf{Z}_{2j}^{st} \\ \mathbf{V}_2^s \mathbf{L}_{21}^{ss} \sum_{j=1}^2 \mathbf{Z}_{1j}^{st} + \mathbf{V}_2^s \mathbf{L}_{22}^{ss} \sum_{j=1}^2 \mathbf{Z}_{2j}^{st} \\ \mathbf{V}_1^r \mathbf{L}_{11}^{rr} \sum_{j=1}^2 \mathbf{Z}_{1j}^{rt} + \mathbf{V}_1^r \mathbf{L}_{12}^{rr} \sum_{j=1}^2 \mathbf{Z}_{2j}^{rt} \\ \mathbf{V}_2^r \mathbf{L}_{21}^{rr} \sum_{j=1}^2 \mathbf{Z}_{1j}^{rt} + \mathbf{V}_2^r \mathbf{L}_{22}^{rr} \sum_{j=1}^2 \mathbf{Z}_{2j}^{rt} \\ 0 \\ 0 \end{pmatrix}$$

此矩阵的目的
的地均为t国

s国向t国出口中间品并
参与GVC生产活动中包
含的s国部门2的增加值

VA_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\begin{aligned}
 \text{rowSums}(\widehat{V}(B-L)\widehat{\text{rowSums}}(Y)) &= \text{rowSums}(\widehat{V}L\text{Am}BY) = \\
 &= \begin{pmatrix} 0 \\ 0 \\ V_1^r L_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) + V_1^r L_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) \\ V_2^r L_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) + V_2^r L_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{sk} Y_i^k \right) \\ V_1^t L_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) + V_1^t L_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) \\ V_2^t L_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) + V_2^t L_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) \end{pmatrix} + \\
 &+ \begin{pmatrix} V_1^s L_{11}^{ss} \sum_{j=1}^2 A_{1j}^{su} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) + V_1^s L_{12}^{ss} \sum_{j=1}^2 A_{2j}^{su} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) \\ V_2^s L_{21}^{ss} \sum_{j=1}^2 A_{1j}^{su} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) + V_2^s L_{22}^{ss} \sum_{j=1}^2 A_{2j}^{su} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{uk} Y_i^k \right) \\ 0 \\ 0 \\ V_1^t L_{11}^{tr} \sum_{j=1}^2 A_{1j}^{tr} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k \right) + V_1^t L_{12}^{tr} \sum_{j=1}^2 A_{2j}^{tr} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k \right) \\ V_2^t L_{21}^{tr} \sum_{j=1}^2 A_{1j}^{tr} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k \right) + V_2^t L_{22}^{tr} \sum_{j=1}^2 A_{2j}^{tr} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{rk} Y_i^k \right) \end{pmatrix} + \\
 &+ \begin{pmatrix} V_1^s L_{11}^{st} \sum_{j=1}^2 A_{1j}^{st} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) + V_1^s L_{12}^{st} \sum_{j=1}^2 A_{2j}^{st} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) \\ V_2^s L_{21}^{st} \sum_{j=1}^2 A_{1j}^{st} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) + V_2^s L_{22}^{st} \sum_{j=1}^2 A_{2j}^{st} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) \\ V_1^r L_{11}^{rt} \sum_{j=1}^2 A_{1j}^{rt} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) + V_1^r L_{12}^{rt} \sum_{j=1}^2 A_{2j}^{rt} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) \\ V_2^r L_{21}^{rt} \sum_{j=1}^2 A_{1j}^{rt} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) + V_2^r L_{22}^{rt} \sum_{j=1}^2 A_{2j}^{rt} \left(\sum_{k=s,i=1}^{t,2} B_{ji}^{tk} Y_i^k \right) \\ 0 \\ 0 \end{pmatrix} \\
 &= \text{rowSums}(\widehat{V}L * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(Y))^T[1:2]))^T)^D) + \text{rowSums}(\widehat{V}L * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(Y))^T[3:4]))^T)^D) \\
 &+ \text{rowSums}(\widehat{V}L * ((\text{rowSums}(\text{Am} * (\text{BrowSums}(Y))^T[5:6]))^T)^D)
 \end{aligned}$$



七：对UIBE的FGY_GVC(即PLy_GVC分母)代码解析—— $V(B-L)Y$ 分解至国家_行业.世界

UIBE的代码：

```
z1 <- diag( VC)%*%(B%*%B-L%*%L)%*%diag(rowSums( Y ))  
z2 <- diag( VC )%*%(B-L)%*%diag(rowSums( Y ))  
PLy_GVC<- colSums(z1)/colSums(z2)
```

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix}$$

$$\hat{V} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix} = \begin{pmatrix} \sum_{u=s}^t Y_1^{su} \\ \sum_{u=s}^t Y_2^{su} \\ \sum_{u=s}^t Y_1^{ru} \\ \sum_{u=s}^t Y_2^{ru} \\ \sum_{u=s}^t Y_1^{tu} \\ \sum_{u=s}^t Y_2^{tu} \end{pmatrix}$$

$$B - L = \begin{pmatrix} B_{11}^{ss} - L_{11}^{ss} & B_{12}^{ss} - L_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} - L_{21}^{ss} & B_{22}^{ss} - L_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} - L_{11}^{rr} & B_{12}^{rr} - L_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} - L_{21}^{rr} & B_{22}^{rr} - L_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} - L_{11}^{tt} & B_{12}^{tt} - L_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} - L_{21}^{tt} & B_{22}^{tt} - L_{22}^{tt} \end{pmatrix}$$

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

$$(\widehat{B-L})_{\text{rowSums}(Y)} = \begin{pmatrix} B_{11}^{ss} - L_{11}^{ss} & B_{12}^{ss} - L_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} - L_{21}^{ss} & B_{22}^{ss} - L_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} - L_{11}^{rr} & B_{12}^{rr} - L_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} - L_{21}^{rr} & B_{22}^{rr} - L_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} - L_{11}^{tt} & B_{12}^{tt} - L_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} - L_{21}^{tt} & B_{22}^{tt} - L_{22}^{tt} \end{pmatrix} \begin{pmatrix} Y_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_2^t \end{pmatrix}$$

$$(\widehat{B-L})_{\text{rowSums}(Y)} = \begin{pmatrix} (B_{11}^{ss} - L_{11}^{ss})Y_1^s & (B_{12}^{ss} - L_{12}^{ss})Y_2^s & B_{11}^{sr}Y_1^r & B_{12}^{sr}Y_2^r & B_{11}^{st}Y_1^t & B_{12}^{st}Y_2^t \\ (B_{21}^{ss} - L_{21}^{ss})Y_1^s & (B_{22}^{ss} - L_{22}^{ss})Y_2^s & B_{21}^{sr}Y_1^r & B_{22}^{sr}Y_2^r & B_{21}^{st}Y_1^t & B_{22}^{st}Y_2^t \\ B_{11}^{rs}Y_1^s & B_{12}^{rs}Y_2^s & (B_{11}^{rr} - L_{11}^{rr})Y_1^r & (B_{12}^{rr} - L_{12}^{rr})Y_2^r & B_{11}^{rt}Y_1^t & B_{12}^{rt}Y_2^t \\ B_{21}^{rs}Y_1^s & B_{22}^{rs}Y_2^s & (B_{21}^{rr} - L_{21}^{rr})Y_1^r & (B_{22}^{rr} - L_{22}^{rr})Y_2^r & B_{21}^{rt}Y_1^t & B_{22}^{rt}Y_2^t \\ B_{11}^{ts}Y_1^s & B_{12}^{ts}Y_2^s & B_{11}^{tr}Y_1^r & B_{12}^{tr}Y_2^r & (B_{11}^{tt} - L_{11}^{tt})Y_1^t & (B_{12}^{tt} - L_{12}^{tt})Y_2^t \\ B_{21}^{ts}Y_1^s & B_{22}^{ts}Y_2^s & B_{21}^{tr}Y_1^r & B_{22}^{tr}Y_2^r & (B_{21}^{tt} - L_{21}^{tt})Y_1^t & (B_{22}^{tt} - L_{22}^{tt})Y_2^t \end{pmatrix}$$

$$\widehat{V}(\widehat{B-L})_{\text{rowSums}(Y)} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \begin{pmatrix} (B_{11}^{ss} - L_{11}^{ss})Y_1^s & (B_{12}^{ss} - L_{12}^{ss})Y_2^s & B_{11}^{sr}Y_1^r & B_{12}^{sr}Y_2^r & B_{11}^{st}Y_1^t & B_{12}^{st}Y_2^t \\ (B_{21}^{ss} - L_{21}^{ss})Y_1^s & (B_{22}^{ss} - L_{22}^{ss})Y_2^s & B_{21}^{sr}Y_1^r & B_{22}^{sr}Y_2^r & B_{21}^{st}Y_1^t & B_{22}^{st}Y_2^t \\ B_{11}^{rs}Y_1^s & B_{12}^{rs}Y_2^s & (B_{11}^{rr} - L_{11}^{rr})Y_1^r & (B_{12}^{rr} - L_{12}^{rr})Y_2^r & B_{11}^{rt}Y_1^t & B_{12}^{rt}Y_2^t \\ B_{21}^{rs}Y_1^s & B_{22}^{rs}Y_2^s & (B_{21}^{rr} - L_{21}^{rr})Y_1^r & (B_{22}^{rr} - L_{22}^{rr})Y_2^r & B_{21}^{rt}Y_1^t & B_{22}^{rt}Y_2^t \\ B_{11}^{ts}Y_1^s & B_{12}^{ts}Y_2^s & B_{11}^{tr}Y_1^r & B_{12}^{tr}Y_2^r & (B_{11}^{tt} - L_{11}^{tt})Y_1^t & (B_{12}^{tt} - L_{12}^{tt})Y_2^t \\ B_{21}^{ts}Y_1^s & B_{22}^{ts}Y_2^s & B_{21}^{tr}Y_1^r & B_{22}^{tr}Y_2^r & (B_{21}^{tt} - L_{21}^{tt})Y_1^t & (B_{22}^{tt} - L_{22}^{tt})Y_2^t \end{pmatrix}$$

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

$$\widehat{V(B-L)rowSums(Y)} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \begin{pmatrix} (B_{11}^{ss} - L_{11}^{ss})Y_1^s & (B_{12}^{ss} - L_{12}^{ss})Y_2^s & B_{11}^{sr}Y_1^r & B_{12}^{sr}Y_2^r & B_{11}^{st}Y_1^t & B_{12}^{st}Y_2^t \\ (B_{21}^{ss} - L_{21}^{ss})Y_1^s & (B_{22}^{ss} - L_{22}^{ss})Y_2^s & B_{21}^{sr}Y_1^r & B_{22}^{sr}Y_2^r & B_{21}^{st}Y_1^t & B_{22}^{st}Y_2^t \\ B_{11}^{rs}Y_1^s & B_{12}^{rs}Y_2^s & (B_{11}^{rr} - L_{11}^{rr})Y_1^r & (B_{12}^{rr} - L_{12}^{rr})Y_2^r & B_{11}^{rt}Y_1^t & B_{12}^{rt}Y_2^t \\ B_{21}^{rs}Y_1^s & B_{22}^{rs}Y_2^s & (B_{21}^{rr} - L_{21}^{rr})Y_1^r & (B_{22}^{rr} - L_{22}^{rr})Y_2^r & B_{21}^{rt}Y_1^t & B_{22}^{rt}Y_2^t \\ B_{11}^{ts}Y_1^s & B_{12}^{ts}Y_2^s & B_{11}^{tr}Y_1^r & B_{12}^{tr}Y_2^r & (B_{11}^{tt} - L_{11}^{tt})Y_1^t & (B_{12}^{tt} - L_{12}^{tt})Y_2^t \\ B_{21}^{ts}Y_1^s & B_{22}^{ts}Y_2^s & B_{21}^{tr}Y_1^r & B_{22}^{tr}Y_2^r & (B_{21}^{tt} - L_{21}^{tt})Y_1^t & (B_{22}^{tt} - L_{22}^{tt})Y_2^t \end{pmatrix}$$

$$\widehat{V(B-L)rowSums(Y)} = \begin{pmatrix} V_1^s(B_{11}^{ss} - L_{11}^{ss})Y_1^s & V_1^s(B_{12}^{ss} - L_{12}^{ss})Y_2^s & V_1^s B_{11}^{sr}Y_1^r & V_1^s B_{12}^{sr}Y_2^r & V_1^s B_{11}^{st}Y_1^t & V_1^s B_{12}^{st}Y_2^t \\ V_2^s(B_{21}^{ss} - L_{21}^{ss})Y_1^s & V_2^s(B_{22}^{ss} - L_{22}^{ss})Y_2^s & V_2^s B_{21}^{sr}Y_1^r & V_2^s B_{22}^{sr}Y_2^r & V_2^s B_{21}^{st}Y_1^t & V_2^s B_{22}^{st}Y_2^t \\ V_1^r B_{11}^{rs}Y_1^s & V_1^r B_{12}^{rs}Y_2^s & V_1^r(B_{11}^{rr} - L_{11}^{rr})Y_1^r & V_1^r(B_{12}^{rr} - L_{12}^{rr})Y_2^r & V_1^r B_{11}^{rt}Y_1^t & V_1^r B_{12}^{rt}Y_2^t \\ V_2^r B_{21}^{rs}Y_1^s & V_2^r B_{22}^{rs}Y_2^s & V_2^r(B_{21}^{rr} - L_{21}^{rr})Y_1^r & V_2^r(B_{22}^{rr} - L_{22}^{rr})Y_2^r & V_2^r B_{21}^{rt}Y_1^t & V_2^r B_{22}^{rt}Y_2^t \\ V_1^t B_{11}^{ts}Y_1^s & V_1^t B_{12}^{ts}Y_2^s & V_1^t B_{11}^{tr}Y_1^r & V_1^t B_{12}^{tr}Y_2^r & V_1^t(B_{11}^{tt} - L_{11}^{tt})Y_1^t & V_1^t(B_{12}^{tt} - L_{12}^{tt})Y_2^t \\ V_2^t B_{21}^{ts}Y_1^s & V_2^t B_{22}^{ts}Y_2^s & V_2^t B_{21}^{tr}Y_1^r & V_2^t B_{22}^{tr}Y_2^r & V_2^t(B_{21}^{tt} - L_{21}^{tt})Y_1^t & V_2^t(B_{22}^{tt} - L_{22}^{tt})Y_2^t \end{pmatrix}$$

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

$$\text{colSums}(\widehat{\hat{V}}(B-L)\widehat{\text{rowSums}(Y)}) = \begin{pmatrix} V_1^s(B_{11}^{ss} - L_{11}^{ss})Y_1^s & V_1^s(B_{12}^{ss} - L_{12}^{ss})Y_2^s & V_1^s B_{11}^{sr} Y_1^r & V_1^s B_{12}^{sr} Y_2^r & V_1^s B_{11}^{st} Y_1^t & V_1^s B_{12}^{st} Y_2^t \\ +V_2^s(B_{21}^{ss} - L_{21}^{ss})Y_1^s & +V_2^s(B_{22}^{ss} - L_{22}^{ss})Y_2^s & +V_2^s B_{21}^{sr} Y_1^r & +V_2^s B_{22}^{sr} Y_2^r & +V_2^s B_{21}^{st} Y_1^t & +V_2^s B_{22}^{st} Y_2^t \\ +V_1^r B_{11}^{rs} Y_1^s & +V_1^r B_{12}^{rs} Y_2^s & +V_1^r (B_{11}^{rr} - L_{11}^{rr})Y_1^r & +V_1^r (B_{12}^{rr} - L_{12}^{rr})Y_2^r & +V_1^r B_{11}^{rt} Y_1^t & +V_1^r B_{12}^{rt} Y_2^t \\ +V_2^r B_{21}^{rs} Y_1^s & +V_2^r B_{22}^{rs} Y_2^s & +V_2^r (B_{21}^{rr} - L_{21}^{rr})Y_1^r & +V_2^r (B_{22}^{rr} - L_{22}^{rr})Y_2^r & +V_2^r B_{21}^{rt} Y_1^t & +V_2^r B_{22}^{rt} Y_2^t \\ +V_1^t B_{11}^{ts} Y_1^s & +V_1^t B_{12}^{ts} Y_2^s & +V_1^t B_{11}^{tr} Y_1^r & +V_1^t B_{12}^{tr} Y_2^r & +V_1^t (B_{11}^{tt} - L_{11}^{tt})Y_1^t & +V_1^t (B_{12}^{tt} - L_{12}^{tt})Y_2^t \\ +V_2^t B_{21}^{ts} Y_1^s & +V_2^t B_{22}^{ts} Y_2^s & +V_2^t B_{21}^{tr} Y_1^r & +V_2^t B_{22}^{tr} Y_2^r & +V_2^t (B_{21}^{tt} - L_{21}^{tt})Y_1^t & +V_2^t (B_{22}^{tt} - L_{22}^{tt})Y_2^t \end{pmatrix}$$

$$\text{colSums}(\widehat{\hat{V}}(B-L)\widehat{\text{rowSums}(Y)}) =$$

$$\left(\sum_{u=s,i=1}^{t,2} V_i^u B_{il}^{us} Y_1^s - \sum_{i=1}^2 V_i^s L_{il}^{ss} Y_1^s \quad \sum_{u=s,i=1}^{t,2} V_i^u B_{i2}^{us} Y_2^s - \sum_{i=1}^2 V_i^s L_{i2}^{ss} Y_2^s \quad \sum_{u=s,i=1}^{t,2} V_i^u B_{il}^{ur} Y_1^r - \sum_{i=1}^2 V_i^r L_{il}^{rr} Y_1^r \quad \sum_{u=s,i=1}^{t,2} V_i^u B_{i2}^{ur} Y_2^r - \sum_{i=1}^2 V_i^r L_{i2}^{rr} Y_2^r \quad \sum_{u=s,i=1}^{t,2} V_i^u B_{il}^{ut} Y_1^t - \sum_{i=1}^2 V_i^t L_{il}^{tt} Y_1^t \quad \sum_{u=s,i=1}^{t,2} V_i^u B_{i2}^{ut} Y_2^t - \sum_{i=1}^2 V_i^t L_{i2}^{tt} Y_2^t \right)$$

根据该结果不易确认目的地是增加值
吸收国还是直接进口国，因此根据公
式 $L\Delta B=B-L$ 对该矩阵作进一步变换

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

以上矩阵表示方法不便确认分解思路，因此采用 $B-L=LA_mB$ 的等式进行替换

$$\begin{cases} AdX + AmX + Y = X \\ BY = X \end{cases} \quad AmX + Y = X - AdX \quad AmBY + Y = (I - Ad)X \quad LAmBY + LY = X \quad LAmBY + LY = BY \quad LAmB + L = B$$

$$L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix}$$

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$LAmB = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix} \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

$$LAmB = \begin{pmatrix} 0 & 0 & \sum_{i=1}^2 L_{1i}^{ss} A_{i1}^{sr} & \sum_{i=1}^2 L_{1i}^{ss} A_{i2}^{sr} & \sum_{i=1}^2 L_{1i}^{ss} A_{i1}^{st} & \sum_{i=1}^2 L_{1i}^{ss} A_{i2}^{st} \\ 0 & 0 & \sum_{i=1}^2 L_{2i}^{ss} A_{i1}^{sr} & \sum_{i=1}^2 L_{2i}^{ss} A_{i2}^{sr} & \sum_{i=1}^2 L_{2i}^{ss} A_{i1}^{st} & \sum_{i=1}^2 L_{2i}^{ss} A_{i2}^{st} \\ \sum_{i=1}^2 L_{1i}^{rr} A_{i1}^{rs} & \sum_{i=1}^2 L_{1i}^{rr} A_{i2}^{rs} & 0 & 0 & \sum_{i=1}^2 L_{1i}^{rr} A_{i1}^{rt} & \sum_{i=1}^2 L_{1i}^{rr} A_{i2}^{rt} \\ \sum_{i=1}^2 L_{2i}^{rr} A_{i1}^{rs} & \sum_{i=1}^2 L_{2i}^{rr} A_{i2}^{rs} & 0 & 0 & \sum_{i=1}^2 L_{2i}^{rr} A_{i1}^{rt} & \sum_{i=1}^2 L_{2i}^{rr} A_{i2}^{rt} \\ \sum_{i=1}^2 L_{1i}^{tt} A_{i1}^{ts} & \sum_{i=1}^2 L_{1i}^{tt} A_{i2}^{ts} & \sum_{i=1}^2 L_{1i}^{tt} A_{i1}^{tr} & \sum_{i=1}^2 L_{1i}^{tt} A_{i2}^{tr} & 0 & 0 \\ \sum_{i=1}^2 L_{2i}^{tt} A_{i1}^{ts} & \sum_{i=1}^2 L_{2i}^{tt} A_{i2}^{ts} & \sum_{i=1}^2 L_{2i}^{tt} A_{i1}^{tr} & \sum_{i=1}^2 L_{2i}^{tt} A_{i2}^{tr} & 0 & 0 \end{pmatrix} \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$LAmB = \begin{pmatrix} \sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us}) & \sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us}) & \sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j1}^{ur}) & \sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ur}) & \sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j1}^{ut}) & \sum_{i=1}^2 L_{1i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ut}) \\ \sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us}) & \sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us}) & \sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j1}^{ur}) & \sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ur}) & \sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j1}^{ut}) & \sum_{i=1}^2 L_{2i}^{ss} (\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ut}) \\ \sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{us}) & \sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{us}) & \sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j1}^{ur}) & \sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur}) & \sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j1}^{ut}) & \sum_{i=1}^2 L_{1i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ut}) \\ \sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{us}) & \sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{us}) & \sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j1}^{ur}) & \sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur}) & \sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j1}^{ut}) & \sum_{i=1}^2 L_{2i}^{rr} (\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ut}) \\ \sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{us}) & \sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{us}) & \sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j1}^{ur}) & \sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ur}) & \sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j1}^{ut}) & \sum_{i=1}^2 L_{1i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut}) \\ \sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{us}) & \sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{us}) & \sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j1}^{ur}) & \sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ur}) & \sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j1}^{ut}) & \sum_{i=1}^2 L_{2i}^{tt} (\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut}) \end{pmatrix}$$

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$$B - L = \begin{pmatrix} B_{11}^{ss} - L_{11}^{ss} & B_{12}^{ss} - L_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} - L_{21}^{ss} & B_{22}^{ss} - L_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} - L_{11}^{rr} & B_{12}^{rr} - L_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} - L_{21}^{rr} & B_{22}^{rr} - L_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} - L_{11}^{tt} & B_{12}^{tt} - L_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} - L_{21}^{tt} & B_{22}^{tt} - L_{22}^{tt} \end{pmatrix}$$

$$LAmB = \begin{pmatrix} \sum_{i=1}^2 L_{1i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us} \right) & \sum_{i=1}^2 L_{1i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us} \right) & \sum_{i=1}^2 L_{1i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ur} \right) & \sum_{i=1}^2 L_{1i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ur} \right) & \sum_{i=1}^2 L_{1i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ut} \right) & \sum_{i=1}^2 L_{1i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ut} \right) \\ \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us} \right) & \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us} \right) & \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ur} \right) & \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ur} \right) & \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ut} \right) & \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ut} \right) \\ \sum_{i=1}^2 L_{1i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{us} \right) & \sum_{i=1}^2 L_{1i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{us} \right) & \sum_{i=1}^2 L_{1i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ur} \right) & \sum_{i=1}^2 L_{1i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur} \right) & \sum_{i=1}^2 L_{1i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ut} \right) & \sum_{i=1}^2 L_{1i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ut} \right) \\ \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{us} \right) & \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{us} \right) & \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ur} \right) & \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur} \right) & \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ut} \right) & \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ut} \right) \\ \sum_{i=1}^2 L_{1i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{us} \right) & \sum_{i=1}^2 L_{1i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{us} \right) & \sum_{i=1}^2 L_{1i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ur} \right) & \sum_{i=1}^2 L_{1i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ur} \right) & \sum_{i=1}^2 L_{1i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ut} \right) & \sum_{i=1}^2 L_{1i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut} \right) \\ \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{us} \right) & \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{us} \right) & \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ur} \right) & \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ur} \right) & \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ut} \right) & \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut} \right) \end{pmatrix}$$

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$$\widehat{V}(B-L)\widehat{\text{rowSums}(Y)} = \widehat{V}(L\widehat{A}\widehat{M}B)\widehat{\text{rowSums}(Y)}$$

$$\left(\begin{array}{cccccc} V_1^s \sum_{i=1}^2 L_{li}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us} \right) Y_1^s & V_1^s \sum_{i=1}^2 L_{li}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us} \right) Y_2^s & V_1^s \sum_{i=1}^2 L_{li}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ur} \right) Y_1^r & V_1^s \sum_{i=1}^2 L_{li}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ur} \right) Y_2^r & V_1^s \sum_{i=1}^2 L_{li}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ut} \right) Y_1^t & V_1^s \sum_{i=1}^2 L_{li}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ut} \right) Y_2^t \\ V_2^s \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us} \right) Y_1^s & V_2^s \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us} \right) Y_2^s & V_2^s \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ur} \right) Y_1^r & V_2^s \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ur} \right) Y_2^r & V_2^s \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ut} \right) Y_1^t & V_2^s \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ut} \right) Y_2^t \\ V_1^r \sum_{i=1}^2 L_{li}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{us} \right) Y_1^s & V_1^r \sum_{i=1}^2 L_{li}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{us} \right) Y_2^s & V_1^r \sum_{i=1}^2 L_{li}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ur} \right) Y_1^r & V_1^r \sum_{i=1}^2 L_{li}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur} \right) Y_2^r & V_1^r \sum_{i=1}^2 L_{li}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ut} \right) Y_1^t & V_1^r \sum_{i=1}^2 L_{li}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ut} \right) Y_2^t \\ V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{us} \right) Y_1^s & V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{us} \right) Y_2^s & V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ur} \right) Y_1^r & V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur} \right) Y_2^r & V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ut} \right) Y_1^t & V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ut} \right) Y_2^t \\ V_1^t \sum_{i=1}^2 L_{li}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{us} \right) Y_1^s & V_1^t \sum_{i=1}^2 L_{li}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{us} \right) Y_2^s & V_1^t \sum_{i=1}^2 L_{li}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ur} \right) Y_1^r & V_1^t \sum_{i=1}^2 L_{li}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ur} \right) Y_2^r & V_1^t \sum_{i=1}^2 L_{li}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ut} \right) Y_1^t & V_1^t \sum_{i=1}^2 L_{li}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut} \right) Y_2^t \\ V_2^t \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{us} \right) Y_1^s & V_2^t \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{us} \right) Y_2^s & V_2^t \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ur} \right) Y_1^r & V_2^t \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ur} \right) Y_2^r & V_2^t \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ut} \right) Y_1^t & V_2^t \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut} \right) Y_2^t \end{array} \right)$$

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

$$\text{colSums}(\widehat{V}(\widehat{B-L})\widehat{\text{rowSums}}(Y)) = \text{colSums}(\widehat{V}(\widehat{LAmB})\widehat{\text{rowSums}}(Y))$$

$$\left(\begin{array}{cccccc} V_1^s \sum_{i=1}^2 L_{li}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us} \right) Y_1^s & V_1^s \sum_{i=1}^2 L_{li}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us} \right) Y_2^s & V_1^s \sum_{i=1}^2 L_{li}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ur} \right) Y_1^r & V_1^s \sum_{i=1}^2 L_{li}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ur} \right) Y_2^r & V_1^s \sum_{i=1}^2 L_{li}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ut} \right) Y_1^t & V_1^s \sum_{i=1}^2 L_{li}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ut} \right) Y_2^t \\ + V_2^s \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us} \right) Y_1^s & + V_2^s \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us} \right) Y_2^s & + V_2^s \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ur} \right) Y_1^r & + V_2^s \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ur} \right) Y_2^r & + V_2^s \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{ut} \right) Y_1^t & + V_2^s \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{ut} \right) Y_2^t \\ + V_1^r \sum_{i=1}^2 L_{li}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{us} \right) Y_1^s & + V_1^r \sum_{i=1}^2 L_{li}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{us} \right) Y_2^s & + V_1^r \sum_{i=1}^2 L_{li}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ur} \right) Y_1^r & + V_1^r \sum_{i=1}^2 L_{li}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur} \right) Y_2^r & + V_1^r \sum_{i=1}^2 L_{li}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ut} \right) Y_1^t & + V_1^r \sum_{i=1}^2 L_{li}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ut} \right) Y_2^t \\ + V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{us} \right) Y_1^s & + V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{us} \right) Y_2^s & + V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ur} \right) Y_1^r & + V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur} \right) Y_2^r & + V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ut} \right) Y_1^t & + V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ut} \right) Y_2^t \\ + V_1^t \sum_{i=1}^2 L_{li}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{us} \right) Y_1^s & + V_1^t \sum_{i=1}^2 L_{li}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{us} \right) Y_2^s & + V_1^t \sum_{i=1}^2 L_{li}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ur} \right) Y_1^r & + V_1^t \sum_{i=1}^2 L_{li}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ur} \right) Y_2^r & + V_1^t \sum_{i=1}^2 L_{li}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ut} \right) Y_1^t & + V_1^t \sum_{i=1}^2 L_{li}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut} \right) Y_2^t \\ + V_2^t \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{us} \right) Y_1^s & + V_2^t \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{us} \right) Y_2^s & + V_2^t \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ur} \right) Y_1^r & + V_2^t \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ur} \right) Y_2^r & + V_2^t \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ut} \right) Y_1^t & + V_2^t \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut} \right) Y_2^t \end{array} \right)$$

使用此分解方法依旧无法识别出目的地是最终吸收地还是直接进口国。根据WWZ（2015）的FVA分解结果，继续采用BAmL=B-L进行转化



FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

以上矩阵表示方法不便确认分解思路，因此采用B-L=LAmB的等式进行替换

$$\begin{cases} \text{AdX} + \text{AmX} + \text{Y} = \text{X} \\ \text{BY} = \text{X} \end{cases} \quad \text{AmX} + \text{Y} = \text{X} - \text{AdX} \quad \text{AmBY} + \text{Y} = (\text{I} - \text{Ad})\text{X} \quad \text{LAmBY} + \text{LY} = \text{X} \quad \text{LAmBY} + \text{LY} = \text{BY} \quad \text{LAmB} + \text{L} = \text{B}$$

$$\text{L} = \text{B} - \text{LAmB} \quad \text{L} = (\text{I} - \text{LAm})\text{B} \quad \text{B} = (\text{I} - \text{LAm})^{-1}\text{L} \quad \text{B} = [\text{I} + (\text{LAm}) + (\text{LAm})(\text{LAm}) + (\text{LAm})(\text{LAm})(\text{LAm}) + (\text{LAm})(\text{LAm})(\text{LAm})(\text{LAm}) + \cdots]\text{L}$$

$$\text{B} = [\text{I} + (\text{LAm})\text{L} + (\text{LAm})(\text{LAm})\text{L} + (\text{LAm})(\text{LAm})(\text{LAm})\text{L} + (\text{LAm})(\text{LAm})(\text{LAm})(\text{LAm})\text{L} + \cdots]$$

$$\text{B} = [\text{I} + \text{L}(\text{AmL}) + \text{L}(\text{AmL})(\text{AmL}) + \text{L}(\text{AmL})(\text{AmL})(\text{AmL}) + \text{L}(\text{AmL})(\text{AmL})(\text{AmL})(\text{AmL}) + \cdots]$$

$$\text{B} = \text{L}[\text{I} + (\text{AmL}) + (\text{AmL})(\text{AmL}) + (\text{AmL})(\text{AmL})(\text{AmL}) + (\text{AmL})(\text{AmL})(\text{AmL})(\text{AmL}) + \cdots]$$

$$\text{B} = \text{L}[\text{I} - (\text{AmL})]^{-1} \quad \text{B}[\text{I} - (\text{AmL})] = \text{L} \quad \text{B} - \text{BAmL} = \text{L} \quad \text{B} - \text{L} = \text{BAmL} \quad \text{B} - \text{L} = \text{BAmL} = \text{LAmB}$$

证明B-L=LAmB=BAmL的另一种方法

$$\text{B}[\text{I} - \text{A}] = \text{I} \rightarrow \text{B} - \text{BA} = \text{I} \rightarrow \text{B} - \text{BAd} - \text{BAm} = \text{I} \rightarrow \text{B}(\text{I} - \text{Ad}) = \text{I} + \text{BAm} \rightarrow \text{B} = (\text{I} - \text{Ad})^{-1} + \text{BAm}(\text{I} - \text{Ad})^{-1} \rightarrow \text{B} = \text{L} + \text{BAmL}$$

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

$$L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix}$$

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$BAmL = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix} \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix}$$

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

$$\text{BAmL} = \left(\begin{array}{cccccc} \sum_{u \neq s, j=1}^{t,2} B_{1j}^{\text{su}} A_{jl}^{\text{us}} & \sum_{u \neq s, j=1}^{t,2} B_{1j}^{\text{su}} A_{j2}^{\text{us}} & \sum_{u \neq r, j=1}^{t,2} B_{1j}^{\text{su}} A_{jl}^{\text{ur}} & \sum_{u \neq r, j=1}^{t,2} B_{1j}^{\text{su}} A_{j2}^{\text{ur}} & \sum_{u \neq t, j=1}^{r,2} B_{1j}^{\text{su}} A_{jl}^{\text{ut}} & \sum_{u \neq t, j=1}^{r,2} B_{1j}^{\text{su}} A_{j2}^{\text{ut}} \\ \sum_{u \neq s, j=1}^{t,2} B_{2j}^{\text{su}} A_{jl}^{\text{us}} & \sum_{u \neq s, j=1}^{t,2} B_{2j}^{\text{su}} A_{j2}^{\text{us}} & \sum_{u \neq r, j=1}^{t,2} B_{2j}^{\text{su}} A_{jl}^{\text{ur}} & \sum_{u \neq r, j=1}^{t,2} B_{2j}^{\text{su}} A_{j2}^{\text{ur}} & \sum_{u \neq t, j=1}^{r,2} B_{2j}^{\text{su}} A_{jl}^{\text{ut}} & \sum_{u \neq t, j=1}^{r,2} B_{2j}^{\text{su}} A_{j2}^{\text{ut}} \\ \sum_{u \neq s, j=1}^{t,2} B_{1j}^{\text{ru}} A_{jl}^{\text{us}} & \sum_{u \neq s, j=1}^{t,2} B_{1j}^{\text{ru}} A_{j2}^{\text{us}} & \sum_{u \neq r, j=1}^{t,2} B_{1j}^{\text{ru}} A_{jl}^{\text{ur}} & \sum_{u \neq r, j=1}^{t,2} B_{1j}^{\text{ru}} A_{j2}^{\text{ur}} & \sum_{u \neq t, j=1}^{r,2} B_{1j}^{\text{ru}} A_{jl}^{\text{ut}} & \sum_{u \neq t, j=1}^{r,2} B_{1j}^{\text{ru}} A_{j2}^{\text{ut}} \\ \sum_{u \neq s, j=1}^{t,2} B_{2j}^{\text{ru}} A_{jl}^{\text{us}} & \sum_{u \neq s, j=1}^{t,2} B_{2j}^{\text{ru}} A_{j2}^{\text{us}} & \sum_{u \neq r, j=1}^{t,2} B_{2j}^{\text{ru}} A_{jl}^{\text{ur}} & \sum_{u \neq r, j=1}^{t,2} B_{2j}^{\text{ru}} A_{j2}^{\text{ur}} & \sum_{u \neq t, j=1}^{r,2} B_{2j}^{\text{ru}} A_{jl}^{\text{ut}} & \sum_{u \neq t, j=1}^{r,2} B_{2j}^{\text{ru}} A_{j2}^{\text{ut}} \\ \sum_{u \neq s, j=1}^{t,2} B_{1j}^{\text{tu}} A_{jl}^{\text{us}} & \sum_{u \neq s, j=1}^{t,2} B_{1j}^{\text{tu}} A_{j2}^{\text{us}} & \sum_{u \neq r, j=1}^{t,2} B_{1j}^{\text{tu}} A_{jl}^{\text{ur}} & \sum_{u \neq r, j=1}^{t,2} B_{1j}^{\text{tu}} A_{j2}^{\text{ur}} & \sum_{u \neq t, j=1}^{r,2} B_{1j}^{\text{tu}} A_{jl}^{\text{ut}} & \sum_{u \neq t, j=1}^{r,2} B_{1j}^{\text{tu}} A_{j2}^{\text{ut}} \\ \sum_{u \neq s, j=1}^{t,2} B_{2j}^{\text{tu}} A_{jl}^{\text{us}} & \sum_{u \neq s, j=1}^{t,2} B_{2j}^{\text{tu}} A_{j2}^{\text{us}} & \sum_{u \neq r, j=1}^{t,2} B_{2j}^{\text{tu}} A_{jl}^{\text{ur}} & \sum_{u \neq r, j=1}^{t,2} B_{2j}^{\text{tu}} A_{j2}^{\text{ur}} & \sum_{u \neq t, j=1}^{r,2} B_{2j}^{\text{tu}} A_{jl}^{\text{ut}} & \sum_{u \neq t, j=1}^{r,2} B_{2j}^{\text{tu}} A_{j2}^{\text{ut}} \end{array} \right) \begin{pmatrix} L_{11}^{\text{ss}} & L_{12}^{\text{ss}} & 0 & 0 & 0 & 0 \\ L_{21}^{\text{ss}} & L_{22}^{\text{ss}} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{\text{rr}} & L_{12}^{\text{rr}} & 0 & 0 \\ 0 & 0 & L_{21}^{\text{rr}} & L_{22}^{\text{rr}} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{\text{tt}} & L_{12}^{\text{tt}} \\ 0 & 0 & 0 & 0 & L_{21}^{\text{tt}} & L_{22}^{\text{tt}} \end{pmatrix}$$

[illegible]

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

$$\widehat{V}(B-L)\text{rowSums}(Y) = \begin{pmatrix} \mathbf{V}_1^s(\mathbf{B}_{11}^{ss} - \mathbf{L}_{11}^{ss})\mathbf{Y}_1^s & \mathbf{V}_1^s(\mathbf{B}_{12}^{ss} - \mathbf{L}_{12}^{ss})\mathbf{Y}_2^s & \mathbf{V}_1^s\mathbf{B}_{11}^{sr}\mathbf{Y}_1^r & \mathbf{V}_1^s\mathbf{B}_{12}^{sr}\mathbf{Y}_2^r & \mathbf{V}_1^s\mathbf{B}_{11}^{st}\mathbf{Y}_1^t & \mathbf{V}_1^s\mathbf{B}_{12}^{st}\mathbf{Y}_2^t \\ \mathbf{V}_2^s(\mathbf{B}_{21}^{ss} - \mathbf{L}_{21}^{ss})\mathbf{Y}_1^s & \mathbf{V}_2^s(\mathbf{B}_{22}^{ss} - \mathbf{L}_{22}^{ss})\mathbf{Y}_2^s & \mathbf{V}_2^s\mathbf{B}_{21}^{sr}\mathbf{Y}_1^r & \mathbf{V}_2^s\mathbf{B}_{22}^{sr}\mathbf{Y}_2^r & \mathbf{V}_2^s\mathbf{B}_{21}^{st}\mathbf{Y}_1^t & \mathbf{V}_2^s\mathbf{B}_{22}^{st}\mathbf{Y}_2^t \\ \mathbf{V}_1^r\mathbf{B}_{11}^{rs}\mathbf{Y}_1^s & \mathbf{V}_1^r\mathbf{B}_{12}^{rs}\mathbf{Y}_2^s & \mathbf{V}_1^r(\mathbf{B}_{11}^{rr} - \mathbf{L}_{11}^{rr})\mathbf{Y}_1^r & \mathbf{V}_1^r(\mathbf{B}_{12}^{rr} - \mathbf{L}_{12}^{rr})\mathbf{Y}_2^r & \mathbf{V}_1^r\mathbf{B}_{11}^{rt}\mathbf{Y}_1^t & \mathbf{V}_1^r\mathbf{B}_{12}^{rt}\mathbf{Y}_2^t \\ \mathbf{V}_2^r\mathbf{B}_{21}^{rs}\mathbf{Y}_1^s & \mathbf{V}_2^r\mathbf{B}_{22}^{rs}\mathbf{Y}_2^s & \mathbf{V}_2^r(\mathbf{B}_{21}^{rr} - \mathbf{L}_{21}^{rr})\mathbf{Y}_1^r & \mathbf{V}_2^r(\mathbf{B}_{22}^{rr} - \mathbf{L}_{22}^{rr})\mathbf{Y}_2^r & \mathbf{V}_2^r\mathbf{B}_{21}^{rt}\mathbf{Y}_1^t & \mathbf{V}_2^r\mathbf{B}_{22}^{rt}\mathbf{Y}_2^t \\ \mathbf{V}_1^t\mathbf{B}_{11}^{ts}\mathbf{Y}_1^s & \mathbf{V}_1^t\mathbf{B}_{12}^{ts}\mathbf{Y}_2^s & \mathbf{V}_1^t\mathbf{B}_{11}^{tr}\mathbf{Y}_1^r & \mathbf{V}_1^t\mathbf{B}_{12}^{tr}\mathbf{Y}_2^r & \mathbf{V}_1^t(\mathbf{B}_{11}^{tt} - \mathbf{L}_{11}^{tt})\mathbf{Y}_1^t & \mathbf{V}_1^t(\mathbf{B}_{12}^{tt} - \mathbf{L}_{12}^{tt})\mathbf{Y}_2^t \\ \mathbf{V}_2^t\mathbf{B}_{21}^{ts}\mathbf{Y}_1^s & \mathbf{V}_2^t\mathbf{B}_{22}^{ts}\mathbf{Y}_2^s & \mathbf{V}_2^t\mathbf{B}_{21}^{tr}\mathbf{Y}_1^r & \mathbf{V}_2^t\mathbf{B}_{22}^{tr}\mathbf{Y}_2^r & \mathbf{V}_2^t(\mathbf{B}_{21}^{tt} - \mathbf{L}_{21}^{tt})\mathbf{Y}_1^t & \mathbf{V}_2^t(\mathbf{B}_{22}^{tt} - \mathbf{L}_{22}^{tt})\mathbf{Y}_2^t \end{pmatrix}$$

由该矩阵可知，主对角线上（标红的分块阵）均为国内增加值，非对角线上（黑色的分块阵）均为国外增加值，因此FGY_GVC实际上是由国外增加值与国内增加值共同组成

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

$$\widehat{V}(B-L)\widehat{\text{rowSums}}(Y) = \begin{pmatrix} V_1^s (B_{11}^{ss} - L_{11}^{ss}) Y_1^s & V_1^s (B_{12}^{ss} - L_{12}^{ss}) Y_2^s & V_1^s B_{11}^{sr} Y_1^r & V_1^s B_{12}^{sr} Y_2^r & V_1^s B_{11}^{st} Y_1^t & V_1^s B_{12}^{st} Y_2^t \\ V_2^s (B_{21}^{ss} - L_{21}^{ss}) Y_1^s & V_2^s (B_{22}^{ss} - L_{22}^{ss}) Y_2^s & V_2^s B_{21}^{sr} Y_1^r & V_2^s B_{22}^{sr} Y_2^r & V_2^s B_{21}^{st} Y_1^t & V_2^s B_{22}^{st} Y_2^t \\ V_1^r B_{11}^{rs} Y_1^s & V_1^r B_{12}^{rs} Y_2^s & V_1^r (B_{11}^{rr} - L_{11}^{rr}) Y_1^r & V_1^r (B_{12}^{rr} - L_{12}^{rr}) Y_2^r & V_1^r B_{11}^{rt} Y_1^t & V_1^r B_{12}^{rt} Y_2^t \\ V_2^r B_{21}^{rs} Y_1^s & V_2^r B_{22}^{rs} Y_2^s & V_2^r (B_{21}^{rr} - L_{21}^{rr}) Y_1^r & V_2^r (B_{22}^{rr} - L_{22}^{rr}) Y_2^r & V_2^r B_{21}^{rt} Y_1^t & V_2^r B_{22}^{rt} Y_2^t \\ V_1^t B_{11}^{ts} Y_1^s & V_1^t B_{12}^{ts} Y_2^s & V_1^t B_{11}^{tr} Y_1^r & V_1^t B_{12}^{tr} Y_2^r & V_1^t (B_{11}^{tt} - L_{11}^{tt}) Y_1^t & V_1^t (B_{12}^{tt} - L_{12}^{tt}) Y_2^t \\ V_2^t B_{21}^{ts} Y_1^s & V_2^t B_{22}^{ts} Y_2^s & V_2^t B_{21}^{tr} Y_1^r & V_2^t B_{22}^{tr} Y_2^r & V_2^t (B_{21}^{tt} - L_{21}^{tt}) Y_1^t & V_2^t (B_{22}^{tt} - L_{22}^{tt}) Y_2^t \end{pmatrix}$$

$$\widehat{V}(B-L)\widehat{\text{rowSums}}(Y) = \widehat{V}(\text{BAmL})\widehat{\text{rowSums}}(Y)$$

$$\begin{pmatrix} V_1^s \sum_{i=1}^2 L_{1i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us} \right) Y_1^s & V_1^s \sum_{i=1}^2 L_{1i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us} \right) Y_2^s & V_1^s \sum_{u \neq r, j=1}^2 B_{1j}^{su} \left(\sum_{i=1}^2 A_{ji}^{ur} L_{i1}^{rr} \right) Y_1^r & V_1^s \sum_{u \neq r, j=1}^2 B_{1j}^{su} \left(\sum_{i=1}^2 A_{ji}^{ur} L_{i2}^{rr} \right) Y_2^r & V_1^s \sum_{u \neq t, j=1}^{r,2} B_{1j}^{su} \left(\sum_{i=1}^2 A_{ji}^{ut} L_{i1}^{tt} \right) Y_1^t & V_1^s \sum_{u \neq t, j=1}^{r,2} B_{1j}^{su} \left(\sum_{i=1}^2 A_{ji}^{ut} L_{i2}^{tt} \right) Y_2^t \\ V_2^s \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{jl}^{us} \right) Y_1^s & V_2^s \sum_{i=1}^2 L_{2i}^{ss} \left(\sum_{u \neq s, j=1}^{t,2} A_{ij}^{su} B_{j2}^{us} \right) Y_2^s & V_2^s \sum_{u \neq r, j=1}^{t,2} B_{2j}^{su} \left(\sum_{i=1}^2 A_{ji}^{ur} L_{i1}^{rr} \right) Y_1^r & V_2^s \sum_{u \neq r, j=1}^{t,2} B_{2j}^{su} \left(\sum_{i=1}^2 A_{ji}^{ur} L_{i2}^{rr} \right) Y_2^r & V_2^s \sum_{u \neq t, j=1}^{r,2} B_{2j}^{su} \left(\sum_{i=1}^2 A_{ji}^{ut} L_{i1}^{tt} \right) Y_1^t & V_2^s \sum_{u \neq t, j=1}^{r,2} B_{2j}^{su} \left(\sum_{i=1}^2 A_{ji}^{ut} L_{i2}^{tt} \right) Y_2^t \\ V_1^r \sum_{u \neq s, j=1}^{t,2} B_{1j}^{ru} \left(\sum_{i=1}^2 A_{ji}^{us} L_{i1}^{ss} \right) Y_1^s & V_1^r \sum_{u \neq s, j=1}^{t,2} B_{1j}^{ru} \left(\sum_{i=1}^2 A_{ji}^{us} L_{i2}^{ss} \right) Y_2^s & V_1^r \sum_{i=1}^2 L_{1i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ur} \right) Y_1^r & V_1^r \sum_{i=1}^2 L_{1i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur} \right) Y_2^r & V_1^r \sum_{u \neq t, j=1}^{r,2} B_{1j}^{ru} \left(\sum_{i=1}^2 A_{ji}^{ut} L_{i1}^{tt} \right) Y_1^t & V_1^r \sum_{u \neq t, j=1}^{r,2} B_{1j}^{ru} \left(\sum_{i=1}^2 A_{ji}^{ut} L_{i2}^{tt} \right) Y_2^t \\ V_2^r \sum_{u \neq s, j=1}^{t,2} B_{2j}^{ru} \left(\sum_{i=1}^2 A_{ji}^{us} L_{i1}^{ss} \right) Y_1^s & V_2^r \sum_{u \neq s, j=1}^{t,2} B_{2j}^{ru} \left(\sum_{i=1}^2 A_{ji}^{us} L_{i2}^{ss} \right) Y_2^s & V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ur} \right) Y_1^r & V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{j2}^{ur} \right) Y_2^r & V_2^r \sum_{u \neq t, j=1}^{r,2} B_{2j}^{ru} \left(\sum_{i=1}^2 A_{ji}^{ut} L_{i1}^{tt} \right) Y_1^t & V_2^r \sum_{u \neq t, j=1}^{r,2} B_{2j}^{ru} \left(\sum_{i=1}^2 A_{ji}^{ut} L_{i2}^{tt} \right) Y_2^t \\ V_1^t \sum_{u \neq s, j=1}^{t,2} B_{1j}^{tu} \left(\sum_{i=1}^2 A_{ji}^{us} L_{i1}^{ss} \right) Y_1^s & V_1^t \sum_{u \neq s, j=1}^{t,2} B_{1j}^{tu} \left(\sum_{i=1}^2 A_{ji}^{us} L_{i2}^{ss} \right) Y_2^s & V_1^t \sum_{u \neq r, j=1}^{t,2} B_{1j}^{tu} \left(\sum_{i=1}^2 A_{ji}^{ur} L_{i1}^{rr} \right) Y_1^r & V_1^t \sum_{u \neq r, j=1}^{t,2} B_{1j}^{tu} \left(\sum_{i=1}^2 A_{ji}^{ur} L_{i2}^{rr} \right) Y_2^r & V_2^r \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq r, j=1}^{t,2} A_{ij}^{ru} B_{jl}^{ur} \right) Y_2^r & V_1^t \sum_{i=1}^2 L_{1i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut} \right) Y_2^t \\ V_2^t \sum_{u \neq s, j=1}^{t,2} B_{2j}^{tu} \left(\sum_{i=1}^2 A_{ji}^{us} L_{i1}^{ss} \right) Y_1^s & V_2^t \sum_{u \neq s, j=1}^{t,2} B_{2j}^{tu} \left(\sum_{i=1}^2 A_{ji}^{us} L_{i2}^{ss} \right) Y_2^s & V_2^t \sum_{u \neq r, j=1}^{t,2} B_{2j}^{tu} \left(\sum_{i=1}^2 A_{ji}^{ur} L_{i1}^{rr} \right) Y_1^r & V_2^t \sum_{u \neq r, j=1}^{t,2} B_{2j}^{tu} \left(\sum_{i=1}^2 A_{ji}^{ur} L_{i2}^{rr} \right) Y_2^r & V_2^t \sum_{i=1}^2 L_{2i}^{rr} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{jl}^{ut} \right) Y_1^t & V_2^t \sum_{i=1}^2 L_{2i}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} A_{ij}^{tu} B_{j2}^{ut} \right) Y_2^t \end{pmatrix}$$

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

$$\text{colSums}(\widehat{V}(B-L)\widehat{\text{rowSums}}(Y)) = \text{colSums}(\widehat{V}(\text{BAmL})\widehat{\text{rowSums}}(Y)) =$$

$$\left(\begin{array}{cc} V_1^s \sum_{u \neq s, j=1}^{t,2} B_{1j}^{su} \left(\sum_{i=1}^2 A_{ji}^{us} L_{il}^{ss} \right) Y_1^s & V_1^s \sum_{u \neq s, j=1}^{t,2} B_{1j}^{su} \left(\sum_{i=1}^2 A_{ji}^{us} L_{i2}^{ss} \right) Y_2^s \\ + V_2^s \sum_{u \neq s, j=1}^{t,2} B_{2j}^{su} \left(\sum_{i=1}^2 A_{ji}^{us} L_{il}^{ss} \right) Y_1^s & + V_2^s \sum_{u \neq s, j=1}^{t,2} B_{2j}^{su} \left(\sum_{i=1}^2 A_{ji}^{us} L_{i2}^{ss} \right) Y_2^s \\ + V_1^r \sum_{u \neq r, j=1}^{t,2} B_{1j}^{ru} \left(\sum_{i=1}^2 A_{ji}^{ur} L_{il}^{ss} \right) Y_1^s & + V_1^r \sum_{u \neq r, j=1}^{t,2} B_{1j}^{ru} \left(\sum_{i=1}^2 A_{ji}^{ur} L_{i2}^{ss} \right) Y_2^s \\ + V_2^r \sum_{u \neq r, j=1}^{t,2} B_{2j}^{ru} \left(\sum_{i=1}^2 A_{ji}^{ur} L_{il}^{ss} \right) Y_1^s & + V_2^r \sum_{u \neq r, j=1}^{t,2} B_{2j}^{ru} \left(\sum_{i=1}^2 A_{ji}^{ur} L_{i2}^{ss} \right) Y_2^s \\ + V_1^t \sum_{u \neq t, j=1}^{t,2} B_{1j}^{tu} \left(\sum_{i=1}^2 A_{ji}^{ut} L_{il}^{ss} \right) Y_1^s & + V_1^t \sum_{u \neq t, j=1}^{t,2} B_{1j}^{tu} \left(\sum_{i=1}^2 A_{ji}^{ut} L_{i2}^{ss} \right) Y_2^s \\ + V_2^t \sum_{u \neq t, j=1}^{t,2} B_{2j}^{tu} \left(\sum_{i=1}^2 A_{ji}^{ut} L_{il}^{ss} \right) Y_1^s & + V_2^t \sum_{u \neq t, j=1}^{t,2} B_{2j}^{tu} \left(\sum_{i=1}^2 A_{ji}^{ut} L_{i2}^{ss} \right) Y_2^s \end{array} \right)$$

u国（u=r、t）对s国出口中间品包含的s国、r国以及t国各部门增加值，这些增加值包含于s国部门2的最终品生产之中。
因此，该项的含义是s国从世界各国进口中间品并用于最终品生产中包含的国内外增加值。

分解思路：

根据WWZ（2015）中关于FVA的分解式可知，对于中间品出口的FVA均以VBAmLY的形式进行分解，因此参考该式子能够得出直接进口国即为目的地。

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

将VBA_mLY的第一列与第二列按照目的地进行拆分

s国从r国进口的中间品中包含的国内增加值

s国从r国进口中间品中包含的r国增加值，即来自直接进口国r国的国外增加值

s国从r国进口中间品中包含的t国增加值，即来自第三国t国的国外增加值

$$\text{colSums}(\widehat{V}(B-L)\widehat{\text{rowSums}}(Y)) = \text{colSums}(\widehat{V}(BAmL)\widehat{\text{rowSums}}(Y)) =$$

$V_1^s \sum_{j=1}^2 B_{1j}^{sr} \left(\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss} \right) Y_1^s$ $+ V_2^s \sum_{j=1}^2 B_{2j}^{sr} \left(\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss} \right) Y_1^s$	$V_1^s \sum_{j=1}^2 B_{1j}^{sr} \left(\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss} \right) Y_2^s$ $+ V_2^s \sum_{j=1}^2 B_{2j}^{sr} \left(\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss} \right) Y_2^s$	$V_1^s \sum_{j=1}^2 B_{1j}^{st} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss} \right) Y_1^s$ $+ V_2^s \sum_{j=1}^2 B_{2j}^{st} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss} \right) Y_1^s$	$V_1^s \sum_{j=1}^2 B_{1j}^{st} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss} \right) Y_2^s$ $+ V_2^s \sum_{j=1}^2 B_{2j}^{st} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss} \right) Y_2^s$
$+ V_1^r \sum_{j=1}^2 B_{1j}^{rr} \left(\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss} \right) Y_1^s$ $+ V_2^r \sum_{j=1}^2 B_{2j}^{rr} \left(\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss} \right) Y_1^s$	$+ V_1^r \sum_{j=1}^2 B_{1j}^{rr} \left(\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss} \right) Y_2^s$ $+ V_2^r \sum_{j=1}^2 B_{2j}^{rr} \left(\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss} \right) Y_2^s$	$+ V_1^r \sum_{j=1}^2 B_{1j}^{rt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss} \right) Y_1^s$ $+ V_2^r \sum_{j=1}^2 B_{2j}^{rt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss} \right) Y_1^s$	$+ V_1^r \sum_{j=1}^2 B_{1j}^{rt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss} \right) Y_2^s$ $+ V_2^r \sum_{j=1}^2 B_{2j}^{rt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss} \right) Y_2^s$
$+ V_1^t \sum_{j=1}^2 B_{1j}^{tr} \left(\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss} \right) Y_1^s$ $+ V_2^t \sum_{j=1}^2 B_{2j}^{tr} \left(\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss} \right) Y_1^s$	$+ V_1^t \sum_{j=1}^2 B_{1j}^{tr} \left(\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss} \right) Y_2^s$ $+ V_2^t \sum_{j=1}^2 B_{2j}^{tr} \left(\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss} \right) Y_2^s$	$+ V_1^t \sum_{j=1}^2 B_{1j}^{tt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss} \right) Y_1^s$ $+ V_2^t \sum_{j=1}^2 B_{2j}^{tt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss} \right) Y_1^s$	$+ V_1^t \sum_{j=1}^2 B_{1j}^{tt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss} \right) Y_2^s$ $+ V_2^t \sum_{j=1}^2 B_{2j}^{tt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss} \right) Y_2^s$

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

$$\begin{aligned}
 \text{colSums}(\widehat{\mathbf{V}}(\mathbf{B}-\mathbf{L})\text{rowSums}(\mathbf{Y}))[1] &= \text{colSums}(\widehat{\mathbf{V}}(\mathbf{BAmL})\text{rowSums}(\mathbf{Y}))[1] \\
 &= \mathbf{V}_1^s \sum_{u \neq s, j=1}^{t,2} \mathbf{B}_{lj}^{su} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{us} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_2^s \sum_{u \neq s, j=1}^{t,2} \mathbf{B}_{2j}^{su} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{us} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_1^r \sum_{u \neq s, j=1}^{t,2} \mathbf{B}_{lj}^{ru} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{us} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_2^r \sum_{u \neq s, j=1}^{t,2} \mathbf{B}_{2j}^{ru} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{us} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_1^t \sum_{u \neq s, j=1}^{t,2} \mathbf{B}_{lj}^{tu} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{us} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_2^t \sum_{u \neq s, j=1}^{t,2} \mathbf{B}_{2j}^{tu} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{us} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s \\
 &= \mathbf{V}_1^s \sum_{j=1}^2 \mathbf{B}_{lj}^{sr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_2^s \sum_{j=1}^2 \mathbf{B}_{2j}^{sr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_1^r \sum_{j=1}^2 \mathbf{B}_{lj}^{rr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_2^r \sum_{j=1}^2 \mathbf{B}_{2j}^{rr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_1^t \sum_{j=1}^2 \mathbf{B}_{lj}^{tr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_2^t \sum_{j=1}^2 \mathbf{B}_{2j}^{tr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s \\
 &\quad + \mathbf{V}_1^s \sum_{j=1}^2 \mathbf{B}_{lj}^{st} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{ts} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_2^s \sum_{j=1}^2 \mathbf{B}_{2j}^{st} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{ts} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_1^r \sum_{j=1}^2 \mathbf{B}_{lj}^{rt} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{ts} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_2^r \sum_{j=1}^2 \mathbf{B}_{2j}^{rt} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{ts} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_1^t \sum_{j=1}^2 \mathbf{B}_{lj}^{tt} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{ts} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_2^t \sum_{j=1}^2 \mathbf{B}_{2j}^{tt} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{ts} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s \\
 &= \sum_{k=s}^t \mathbf{V}_1^k \left[\sum_{j=1}^2 \mathbf{B}_{lj}^{kr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) \right] \mathbf{Y}_1^s + \sum_{k=s}^t \mathbf{V}_2^k \left[\sum_{j=1}^2 \mathbf{B}_{2j}^{kr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) \right] \mathbf{Y}_1^s + \sum_{k=s}^t \mathbf{V}_1^k \left[\sum_{j=1}^2 \mathbf{B}_{lj}^{kt} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{ts} \mathbf{L}_{il}^{ss} \right) \right] \mathbf{Y}_1^s + \sum_{k=s}^t \mathbf{V}_2^k \left[\sum_{j=1}^2 \mathbf{B}_{2j}^{kt} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{ts} \mathbf{L}_{il}^{ss} \right) \right] \mathbf{Y}_1^s
 \end{aligned}$$

对上式中的蓝色公式采用 $\mathbf{BAmL}=\mathbf{LAmB}$ 进行转化

$$\sum_{u \neq s, j=1}^{t,2} \mathbf{B}_{lj}^{tu} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{us} \mathbf{L}_{il}^{ss} \right) = \sum_{j=1}^2 \mathbf{B}_{lj}^{tr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) + \sum_{j=1}^2 \mathbf{B}_{lj}^{tt} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{ts} \mathbf{L}_{il}^{ss} \right) = \sum_{i=1}^2 \mathbf{L}_{li}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} \mathbf{A}_{ij}^{tu} \mathbf{B}_{jl}^{us} \right) = \sum_{i=1}^2 \mathbf{L}_{li}^{tt} \left(\sum_{j=1}^2 \mathbf{A}_{ij}^{tr} \mathbf{B}_{jl}^{rs} \right) + \sum_{i=1}^2 \mathbf{L}_{li}^{tt} \left(\sum_{j=1}^2 \mathbf{A}_{ij}^{ts} \mathbf{B}_{jl}^{ss} \right)$$

$$\mathbf{V}_1^t \sum_{j=1}^2 \mathbf{B}_{lj}^{tr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_1^t \sum_{j=1}^2 \mathbf{B}_{lj}^{tt} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{ts} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s$$

该式子表示r国对s国出口中间品并被s国用于生产最终品中包含的第三国t国的FVA

$$= \mathbf{V}_1^t \sum_{u \neq s, j=1}^{t,2} \mathbf{B}_{lj}^{tu} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{us} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s = \mathbf{V}_1^t \sum_{j=1}^2 \mathbf{B}_{lj}^{tr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \mathbf{V}_1^t \sum_{j=1}^2 \mathbf{B}_{lj}^{tt} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{ts} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s$$

$$= \mathbf{V}_1^t \sum_{i=1}^2 \mathbf{L}_{li}^{tt} \left(\sum_{u \neq t, j=1}^{r,2} \mathbf{A}_{ij}^{tu} \mathbf{B}_{jl}^{us} \right) \mathbf{Y}_1^s = \mathbf{V}_1^t \sum_{i=1}^2 \mathbf{L}_{li}^{tt} \left(\sum_{j=1}^2 \mathbf{A}_{ij}^{tr} \mathbf{B}_{jl}^{rs} \right) \mathbf{Y}_1^s + \mathbf{V}_1^t \sum_{i=1}^2 \mathbf{L}_{li}^{tt} \left(\sum_{j=1}^2 \mathbf{A}_{ij}^{ts} \mathbf{B}_{jl}^{ss} \right) \mathbf{Y}_1^s$$

该式子表示t国对s国出口中间品并被s国用于最终品生产中包含的t国增加值，即t国中间品出口的DVA

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

[illegible]

[illegible]

八：FGY_GVC(即PLy_GVC分母)分解至国家_行业.国家——对V(B-L)Y分解

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V(B-L)Y分解至国家_行业.国家的代码： VB<-diag(VC)%*%B
VB_3<-array(0,dim=c(GN,GN))
for (ii in 1:G) {
  m=1+N*(ii-1) ; n=N+(ii-1)*N
  VB_1<-t(as.matrix(colSums(VB[m:n,])))
  VB_2<-matrix(VB_1,nrow=N,ncol=GN,byrow=TRUE)
  VB_3[m:n,1:GN]<-VB_2[1:N,]
}
AmL<-Am%*%L
VBAmL_2<-array(0,dim=c(GN,GN))
VBAmLY_back<-array(0,dim=c(GN*G,1))
VBAmLY_5<-array(0,dim=c(GN,1))
for (i in 1:G) {
  m=1+N*(i-1) ; n=N+(i-1)*N
  mm=1+GN*(i-1) ; nn=GN+(i-1)*GN
  AmL_1<-t(array(AmL[,m:n],dim=c(GN,GN)))
  VBAmL<-VB_3*AmL_1
  YY<-as.matrix(rowSums(Y))
  YY_1<-matrix(YY[m:n,],nrow=GN,ncol=GN,byrow = FALSE)
  VBAmLY<-VBAmL*YY_1
  for (jj in 1:G){
    ss=1+N*(jj-1) ; uu=N+(jj-1)*N
    VBAmLY_2<-as.matrix(rowSums(VBAmLY[,ss:uu]))
    VBAmLY_3<-VBAmLY_2
    dim(VBAmLY_3)<-c(N,G)
    VBAmLY_4<-as.matrix(rowSums(VBAmLY_3))
    VBAmLY_5[ss:uu,1]<-VBAmLY_4[,1]
  }
  VBAmLY_back[mm:nn,1]<-VBAmLY_5[1:GN,1]
}
```

FGY_GVC分解至国家_行业.世界——V(B-L)Y分解

将VBA_mLY的第一列与第二列按照目的地进行拆分

$$\text{colSums}(\widehat{V}(B-L)\widehat{\text{rowSums}}(Y)) = \text{colSums}(\widehat{V}(\text{BA}_m\text{L})\widehat{\text{rowSums}}(Y)) =$$

$$\begin{pmatrix} V_1^s \sum_{j=1}^2 B_{1j}^{sr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss}) Y_1^s & V_1^s \sum_{j=1}^2 B_{1j}^{sr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss}) Y_2^s & V_1^s \sum_{j=1}^2 B_{1j}^{st} (\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss}) Y_1^s & V_1^s \sum_{j=1}^2 B_{1j}^{st} (\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss}) Y_2^s \\ + V_2^s \sum_{j=1}^2 B_{2j}^{sr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss}) Y_1^s & + V_2^s \sum_{j=1}^2 B_{2j}^{sr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss}) Y_2^s & + V_2^s \sum_{j=1}^2 B_{2j}^{st} (\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss}) Y_1^s & + V_2^s \sum_{j=1}^2 B_{2j}^{st} (\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss}) Y_2^s \\ + V_1^r \sum_{j=1}^2 B_{1j}^{rr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss}) Y_1^s & + V_1^r \sum_{j=1}^2 B_{1j}^{rr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss}) Y_2^s & + V_1^r \sum_{j=1}^2 B_{1j}^{rt} (\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss}) Y_1^s & + V_1^r \sum_{j=1}^2 B_{1j}^{rt} (\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss}) Y_2^s \\ + V_2^r \sum_{j=1}^2 B_{2j}^{rr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss}) Y_1^s & + V_2^r \sum_{j=1}^2 B_{2j}^{rr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss}) Y_2^s & + V_2^r \sum_{j=1}^2 B_{2j}^{rt} (\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss}) Y_1^s & + V_2^r \sum_{j=1}^2 B_{2j}^{rt} (\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss}) Y_2^s \\ + V_1^t \sum_{j=1}^2 B_{1j}^{tr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss}) Y_1^s & + V_1^t \sum_{j=1}^2 B_{1j}^{tr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss}) Y_2^s & + V_1^t \sum_{j=1}^2 B_{1j}^{tt} (\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss}) Y_1^s & + V_1^t \sum_{j=1}^2 B_{1j}^{tt} (\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss}) Y_2^s \\ + V_2^t \sum_{j=1}^2 B_{2j}^{tr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss}) Y_1^s & + V_2^t \sum_{j=1}^2 B_{2j}^{tr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss}) Y_2^s & + V_2^t \sum_{j=1}^2 B_{2j}^{tt} (\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss}) Y_1^s & + V_2^t \sum_{j=1}^2 B_{2j}^{tt} (\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss}) Y_2^s \end{pmatrix}$$

含义解析:

t国对s国出口中间品中包含的r国各部门的国内增加值+r国对s国出口中间品中包含的直接进口国s国各部门的国外增加值+r国对s国出口中间品中包含的来自第三国t国各部门的国外增加值。

$$\text{colSums}(\widehat{V}(B-L)\widehat{\text{rowSums}}(Y)) = \text{colSums}(\widehat{V}(\text{BA}_m\text{L})\widehat{\text{rowSums}}(Y))$$

$$= \begin{pmatrix} \sum_{k=s}^t V_1^k \sum_{j=1}^2 B_{1j}^{kr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss}) Y_1^s & \sum_{k=s}^t V_1^k \sum_{j=1}^2 B_{1j}^{kr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss}) Y_2^s & \sum_{k=s}^t V_1^k \sum_{j=1}^2 B_{1j}^{kt} (\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss}) Y_1^s & \sum_{k=s}^t V_1^k \sum_{j=1}^2 B_{1j}^{kt} (\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss}) Y_2^s \\ + \sum_{k=s}^t V_2^k \sum_{j=1}^2 B_{2j}^{kr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss}) Y_1^s & + \sum_{k=s}^t V_2^k \sum_{j=1}^2 B_{2j}^{kr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss}) Y_2^s & + \sum_{k=s}^t V_2^k \sum_{j=1}^2 B_{2j}^{kt} (\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss}) Y_1^s & + \sum_{k=s}^t V_2^k \sum_{j=1}^2 B_{2j}^{kt} (\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss}) Y_2^s \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i1}^{ss}) Y_1^s & \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kr} (\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss}) Y_2^s & \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kt} (\sum_{i=1}^2 A_{ji}^{ts} L_{i1}^{ss}) Y_1^s & \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kt} (\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss}) Y_2^s \end{pmatrix}$$

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$$L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix}$$

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$\hat{V}B = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} = \begin{pmatrix} V_1^s B_{11}^{ss} & V_1^s B_{12}^{ss} & V_1^s B_{11}^{sr} & V_1^s B_{12}^{sr} & V_1^s B_{11}^{st} & V_1^s B_{12}^{st} \\ V_2^s B_{21}^{ss} & V_2^s B_{22}^{ss} & V_2^s B_{21}^{sr} & V_2^s B_{22}^{sr} & V_2^s B_{21}^{st} & V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} & V_1^r B_{12}^{rs} & V_1^r B_{11}^{rr} & V_1^r B_{12}^{rr} & V_1^r B_{11}^{rt} & V_1^r B_{12}^{rt} \\ V_2^r B_{21}^{rs} & V_2^r B_{22}^{rs} & V_2^r B_{21}^{rr} & V_2^r B_{22}^{rr} & V_2^r B_{21}^{rt} & V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} & V_1^t B_{12}^{ts} & V_1^t B_{11}^{tr} & V_1^t B_{12}^{tr} & V_1^t B_{11}^{tt} & V_1^t B_{12}^{tt} \\ V_2^t B_{21}^{ts} & V_2^t B_{22}^{ts} & V_2^t B_{21}^{tr} & V_2^t B_{22}^{tr} & V_2^t B_{21}^{tt} & V_2^t B_{22}^{tt} \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix}$$

$$\text{rowSums}(Y)[1:2,] = \begin{pmatrix} Y_1^s \\ Y_2^s \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\overline{\text{rowSums}(Y)[1:2,]} = \begin{pmatrix} Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$AmL = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix} \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{sr} L_{j1}^{rr} & \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} & \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} \\ 0 & 0 & \sum_{j=1}^2 A_{2j}^{sr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} & \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} & \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & 0 & 0 & \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} & \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{tr} L_{j1}^{rr} & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & 0 & 0 \\ \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{tr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} & 0 & 0 \end{pmatrix}$$

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$$\text{AmL}[1:2] = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} \\ \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} \\ \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} \\ \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \end{pmatrix}$$

$$\overline{\text{AmL}[1:2]} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} \\ \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} \\ \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} \\ \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \end{pmatrix}$$

$$(\overline{\text{AmL}[1:2]})^T = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \end{pmatrix}$$

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$$\text{colSums}(\hat{V}B[m:n,]) = \begin{pmatrix} V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \end{pmatrix}$$

将VB矩阵的第一行与第二行加总、第三
行与第四行加总、第五行与第六行加总

$$\overline{\text{colSums}(\hat{V}B[m:n,])} = \begin{pmatrix} V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\overline{\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\overline{\text{AmL}}[,1:2])^T} =$$

$$\begin{pmatrix} V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \end{pmatrix} * \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[1:2])^T$

$$= \begin{pmatrix} 0 & 0 & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} & V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} & V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} \\ 0 & 0 & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \\ 0 & 0 & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} & V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} & V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} \\ 0 & 0 & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \\ 0 & 0 & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} & V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} & V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} \\ 0 & 0 & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\overline{\text{colSums}(\widehat{\text{VB}}[m:n]) * (\text{AmL}[1:2])^T * \overline{\text{rowSums}(\text{Y})[1:2]} =$$

$$\begin{pmatrix} 0 & 0 & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} & V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} & V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} \\ 0 & 0 & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \\ 0 & 0 & V_1^r B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} + V_2^r B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} & V_1^r B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} + V_2^r B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} & V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} \\ 0 & 0 & V_1^r B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} + V_2^r B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & V_1^r B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} + V_2^r B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \\ 0 & 0 & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} & V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} & V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} \\ 0 & 0 & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \end{pmatrix}$$

$$* \begin{pmatrix} Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\overline{\text{colSums}(\widehat{VB}[m:n,]) * (\overline{\text{AmL}[1:2]})^T * \overline{\text{rowSums}(Y)[1:2,]}} =$$

$$\begin{pmatrix} 0 & 0 & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s & V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s & V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s \\ 0 & 0 & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s & V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s & V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \\ 0 & 0 & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s & V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s & V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s \\ 0 & 0 & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s & V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s & V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \\ 0 & 0 & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s & V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s & V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s \\ 0 & 0 & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s & V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s & V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\text{rowSums}(\overline{\widehat{\text{VB}}[m:n,]} * (\overline{\text{AmL}[1:2]})^T * \overline{\text{rowSums}(\text{Y})[1:2,]}[m:n]) =$$

$$\begin{pmatrix} 0 & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s \\ 0 & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \\ 0 & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s \\ 0 & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \\ 0 & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s \\ 0 & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \end{pmatrix}$$

此矩阵是将上一页的矩阵的第一列与第二列相加、第三列与第四列相加、第五列与第六列相加后得到的

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$$\text{rowSums}(\overline{(\text{colSums}(\widehat{VB}[m:n,]) * (\overline{AmL[1:2]})^T * \overline{\text{rowSums}(Y)[1:2,]})[m:n]}), [2] = \left(\begin{array}{l} V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s \\ V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s \\ V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s \\ V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s \\ V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s \\ V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s \end{array} \right)$$

$$\dim(\text{rowSums}(\overline{(\text{colSums}(\widehat{VB}[m:n,]) * (\overline{AmL[1:2]})^T * \overline{\text{rowSums}(Y)[1:2,]})[m:n]}), [2]) = c(N, G) =$$

$$\left(\begin{array}{l} V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s \\ V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s \\ V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s \\ V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s \\ V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s \\ V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s \end{array} \right)^T$$

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$$\text{rowSums}(\text{dim}(\text{rowSums}((\text{colSums}(\widehat{\mathbf{V}}\mathbf{B}[m:n,]) * (\mathbf{A}\mathbf{m}\mathbf{L}[1:2])^T * \text{rowSums}(\mathbf{Y})[1:2,])[m:n])[2]) = \mathbf{c}(\mathbf{N}, \mathbf{G})) =$$

$$\left(\begin{array}{cc} \mathbf{V}_1^s \mathbf{B}_{11}^{sr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rs} \mathbf{L}_{jl}^{ss} \mathbf{Y}_1^s + \mathbf{V}_2^s \mathbf{B}_{21}^{sr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rs} \mathbf{L}_{jl}^{ss} \mathbf{Y}_1^s + \mathbf{V}_1^s \mathbf{B}_{12}^{sr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rs} \mathbf{L}_{jl}^{ss} \mathbf{Y}_1^s + \mathbf{V}_2^s \mathbf{B}_{22}^{sr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rs} \mathbf{L}_{jl}^{ss} \mathbf{Y}_1^s & \mathbf{V}_1^s \mathbf{B}_{11}^{sr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rs} \mathbf{L}_{j2}^{ss} \mathbf{Y}_2^s + \mathbf{V}_2^s \mathbf{B}_{21}^{sr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rs} \mathbf{L}_{j2}^{ss} \mathbf{Y}_2^s + \mathbf{V}_1^s \mathbf{B}_{12}^{sr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rs} \mathbf{L}_{j2}^{ss} \mathbf{Y}_2^s + \mathbf{V}_2^s \mathbf{B}_{22}^{sr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rs} \mathbf{L}_{j2}^{ss} \mathbf{Y}_2^s \\ + \mathbf{V}_1^r \mathbf{B}_{11}^{rr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rs} \mathbf{L}_{jl}^{ss} \mathbf{Y}_1^s + \mathbf{V}_2^r \mathbf{B}_{21}^{rr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rs} \mathbf{L}_{jl}^{ss} \mathbf{Y}_1^s + \mathbf{V}_1^r \mathbf{B}_{12}^{rr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rs} \mathbf{L}_{jl}^{ss} \mathbf{Y}_1^s + \mathbf{V}_2^r \mathbf{B}_{22}^{rr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rs} \mathbf{L}_{jl}^{ss} \mathbf{Y}_1^s & + \mathbf{V}_1^r \mathbf{B}_{11}^{rr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rs} \mathbf{L}_{j2}^{ss} \mathbf{Y}_2^s + \mathbf{V}_2^r \mathbf{B}_{21}^{rr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rs} \mathbf{L}_{j2}^{ss} \mathbf{Y}_2^s + \mathbf{V}_1^r \mathbf{B}_{12}^{rr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rs} \mathbf{L}_{j2}^{ss} \mathbf{Y}_2^s + \mathbf{V}_2^r \mathbf{B}_{22}^{rr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rs} \mathbf{L}_{j2}^{ss} \mathbf{Y}_2^s \\ + \mathbf{V}_1^t \mathbf{B}_{11}^{tr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rs} \mathbf{L}_{jl}^{ss} \mathbf{Y}_1^s + \mathbf{V}_2^t \mathbf{B}_{21}^{tr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rs} \mathbf{L}_{jl}^{ss} \mathbf{Y}_1^s + \mathbf{V}_1^t \mathbf{B}_{12}^{tr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rs} \mathbf{L}_{jl}^{ss} \mathbf{Y}_1^s + \mathbf{V}_2^t \mathbf{B}_{22}^{tr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rs} \mathbf{L}_{jl}^{ss} \mathbf{Y}_1^s & + \mathbf{V}_1^t \mathbf{B}_{11}^{tr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rs} \mathbf{L}_{j2}^{ss} \mathbf{Y}_2^s + \mathbf{V}_2^t \mathbf{B}_{21}^{tr} \sum_{j=1}^2 \mathbf{A}_{1j}^{rs} \mathbf{L}_{j2}^{ss} \mathbf{Y}_2^s + \mathbf{V}_1^t \mathbf{B}_{12}^{tr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rs} \mathbf{L}_{j2}^{ss} \mathbf{Y}_2^s + \mathbf{V}_2^t \mathbf{B}_{22}^{tr} \sum_{j=1}^2 \mathbf{A}_{2j}^{rs} \mathbf{L}_{j2}^{ss} \mathbf{Y}_2^s \end{array} \right)^T$$

$$\text{rowSums}(\text{dim}(\text{rowSums}((\text{colSums}(\widehat{\mathbf{V}}\mathbf{B}[m:n,]) * (\mathbf{A}\mathbf{m}\mathbf{L}[1:2])^T * \text{rowSums}(\mathbf{Y})[1:2,])[m:n])[2]) = \mathbf{c}(\mathbf{N}, \mathbf{G}))$$

$$= \left(\sum_{k=s}^t \mathbf{V}_1^k \sum_{j=1}^2 \mathbf{B}_{1j}^{kr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s + \sum_{k=s}^t \mathbf{V}_2^k \sum_{j=1}^2 \mathbf{B}_{2j}^{kr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s \quad \sum_{k=s}^t \mathbf{V}_1^k \sum_{j=1}^2 \mathbf{B}_{1j}^{kr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{i2}^{ss} \right) \mathbf{Y}_2^s + \sum_{k=s}^t \mathbf{V}_2^k \sum_{j=1}^2 \mathbf{B}_{2j}^{kr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{i2}^{ss} \right) \mathbf{Y}_2^s \right)^T = \left(\sum_{k=s, m=1}^{t, 2} \mathbf{V}_m^k \sum_{j=1}^2 \mathbf{B}_{mj}^{kr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s \quad \sum_{k=s, m=1}^{t, 2} \mathbf{V}_m^k \sum_{j=1}^2 \mathbf{B}_{mj}^{kr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{i2}^{ss} \right) \mathbf{Y}_2^s \right)^T$$

含义说明:

s国从r国进口中间品并用于s国部门1参与GVC生产活动中包含的r国的增加值以及其他第三国的增加值

将s国从r国进口中间品的结果放入空矩阵

此矩阵的出口国为r国，直接进口国为s国，并且s国亦是目的地

$$\mathbf{VBLAmY_back}^s =$$

$$\left(\begin{array}{cc} 0 & 0 \quad \sum_{k=s, m=1}^{t, 2} \mathbf{V}_m^k \sum_{j=1}^2 \mathbf{B}_{mj}^{kr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{il}^{ss} \right) \mathbf{Y}_1^s \quad \sum_{k=s, m=1}^{t, 2} \mathbf{V}_m^k \sum_{j=1}^2 \mathbf{B}_{mj}^{kr} \left(\sum_{i=1}^2 \mathbf{A}_{ji}^{rs} \mathbf{L}_{i2}^{ss} \right) \mathbf{Y}_2^s \quad \text{export}^{st} \quad \text{export}^{st} \end{array} \right)^T$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\text{rowSums}(\overline{(\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\overline{\text{AmL}}[1:2])^T * \overline{\text{rowSums}(\text{Y})[1:2,])})[m:n]) =$$

$$\left(\begin{array}{cc} 0 & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s \\ 0 & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \\ 0 & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s \\ 0 & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \\ 0 & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} Y_1^s + V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} Y_1^s & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s \\ 0 & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} Y_2^s + V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} Y_2^s & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \end{array} \right)$$

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$$\text{rowSums}(\overline{(\text{colSums}(\widehat{VB}[m:n,]) * (\overline{AmL[1:2]})^T * \overline{\text{rowSums}(Y)[1:2,]})[m:n]}), [3]) = \begin{pmatrix} V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s \\ V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \\ V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s \\ V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \\ V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s \\ V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \end{pmatrix}$$

$$\dim(\text{rowSums}(\overline{(\text{colSums}(\widehat{VB}[m:n,]) * (\overline{AmL[1:2]})^T * \overline{\text{rowSums}(Y)[1:2,]})[m:n]}), [3]) = c(N, G) =$$

$$\begin{pmatrix} V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \\ V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \\ V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \end{pmatrix}^T$$

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$$\text{rowSums}(\text{dim}(\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[1:2])^T * \text{rowSums}(\text{Y})[1:2,])[m:n])[3]) = c(N, G)) =$$

$$\left(\begin{array}{cc} V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \\ + V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s & + V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \\ + V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} Y_1^s + V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} Y_1^s & + V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} Y_2^s + V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} Y_2^s \end{array} \right)^T$$

$$\text{rowSums}(\text{dim}(\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[1:2])^T * \text{rowSums}(\text{Y})[1:2,])[m:n])[3]) = c(N, G))$$

$$= \left(\sum_{k=s}^t V_1^k \sum_{j=1}^2 B_{1j}^{kt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{il}^{ss} \right) Y_1^s + \sum_{k=s}^t V_2^k \sum_{j=1}^2 B_{2j}^{kt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{il}^{ss} \right) Y_1^s \quad \sum_{k=s}^t V_1^k \sum_{j=1}^2 B_{1j}^{kt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss} \right) Y_2^s + \sum_{k=s}^t V_2^k \sum_{j=1}^2 B_{2j}^{kt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss} \right) Y_2^s \right)^T = \left(\sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{il}^{ss} \right) Y_1^s \quad \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss} \right) Y_2^s \right)^T$$

含义说明：
s国从t国进口中间品并用于s国部门1参与GVC生产活动中包含的t国的增加值以及其他第三国的增加值
此矩阵的出口国为t国，直接进口国为s国，并且s国亦是目的地

将s国从t国进口中间品的结果放入空矩阵

$$\text{VBLAmY_back}^s =$$

$$\left(\begin{array}{cccc} 0 & 0 & \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kr} \left(\sum_{i=1}^2 A_{ji}^{rs} L_{il}^{ss} \right) Y_1^s & \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kr} \left(\sum_{i=1}^2 A_{ji}^{rs} L_{i2}^{ss} \right) Y_2^s \\ \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{il}^{ss} \right) Y_1^s & \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss} \right) Y_2^s & \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{il}^{ss} \right) Y_1^s & \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kt} \left(\sum_{i=1}^2 A_{ji}^{ts} L_{i2}^{ss} \right) Y_2^s \end{array} \right)^T$$

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$$L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix}$$

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$\hat{V}B = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} = \begin{pmatrix} V_1^s B_{11}^{ss} & V_1^s B_{12}^{ss} & V_1^s B_{11}^{sr} & V_1^s B_{12}^{sr} & V_1^s B_{11}^{st} & V_1^s B_{12}^{st} \\ V_2^s B_{21}^{ss} & V_2^s B_{22}^{ss} & V_2^s B_{21}^{sr} & V_2^s B_{22}^{sr} & V_2^s B_{21}^{st} & V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} & V_1^r B_{12}^{rs} & V_1^r B_{11}^{rr} & V_1^r B_{12}^{rr} & V_1^r B_{11}^{rt} & V_1^r B_{12}^{rt} \\ V_2^r B_{21}^{rs} & V_2^r B_{22}^{rs} & V_2^r B_{21}^{rr} & V_2^r B_{22}^{rr} & V_2^r B_{21}^{rt} & V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} & V_1^t B_{12}^{ts} & V_1^t B_{11}^{tr} & V_1^t B_{12}^{tr} & V_1^t B_{11}^{tt} & V_1^t B_{12}^{tt} \\ V_2^t B_{21}^{ts} & V_2^t B_{22}^{ts} & V_2^t B_{21}^{tr} & V_2^t B_{22}^{tr} & V_2^t B_{21}^{tt} & V_2^t B_{22}^{tt} \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix}$$

$$\text{rowSums}(Y)[3:4,] = \begin{pmatrix} 0 \\ 0 \\ Y_1^r \\ Y_2^r \\ 0 \\ 0 \end{pmatrix}$$

$$\overline{\text{rowSums}(Y)[3:4,]} = \begin{pmatrix} Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$AmL = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix} \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{sr} L_{j1}^{rr} & \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} & \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} \\ 0 & 0 & \sum_{j=1}^2 A_{2j}^{sr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} & \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} & \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & 0 & 0 & \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} & \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{tr} L_{j1}^{rr} & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & 0 & 0 \\ \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{tr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} & 0 & 0 \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\overline{\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\overline{\text{AmL}}, 3:4)^T =$$

$$\begin{pmatrix} V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \end{pmatrix} * \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{sr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j1}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j1}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j1}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j1}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j1}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j1}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \end{pmatrix}$$

$$= \begin{pmatrix} V_1^S B_{11}^{SS} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} + V_2^S B_{21}^{SS} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} & V_1^S B_{12}^{SS} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} + V_2^S B_{22}^{SS} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & V_1^S B_{11}^{St} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} + V_2^S B_{21}^{St} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} & V_1^S B_{12}^{St} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} + V_2^S B_{22}^{St} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ V_1^S B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} + V_2^S B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & V_1^S B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} + V_2^S B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & V_1^S B_{11}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} + V_2^S B_{21}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & V_1^S B_{12}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} + V_2^S B_{22}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} & V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} & V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} & V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} & V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\overline{\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[3:4])^T * \overline{\text{rowSums}(Y)[3:4,]}} =$$

$$\begin{pmatrix} V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{j1}^{rr} + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{j1}^{rr} & V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{j1}^{rr} + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{j1}^{rr} & 0 & 0 & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{j1}^{rr} + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{j1}^{rr} & V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{j1}^{rr} + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{j1}^{rr} \\ V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{j1}^{rr} + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{j1}^{rr} & V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{j1}^{rr} + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{j1}^{rr} & 0 & 0 & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{j1}^{rr} + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{j1}^{rr} & V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{j1}^{rr} + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{j1}^{rr} \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{j1}^{rr} + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{j1}^{rr} & V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{j1}^{rr} + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{j1}^{rr} & 0 & 0 & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{j1}^{rr} + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{j1}^{rr} & V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{j1}^{rr} + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{j1}^{rr} \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \end{pmatrix}$$

$$* \begin{pmatrix} Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[3:4])^T * \text{rowSums}(\text{Y})[3:4,] =$$

$$\begin{pmatrix} V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{\pi} Y_1^r + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{\pi} Y_1^r & V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{\pi} Y_1^r + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{\pi} Y_1^r & 0 & 0 & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{\pi} Y_1^r + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{\pi} Y_1^r & V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{\pi} Y_1^r + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{\pi} Y_1^r \\ V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{\pi} Y_2^r + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{\pi} Y_2^r & V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{\pi} Y_2^r + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{\pi} Y_2^r & 0 & 0 & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{\pi} Y_2^r + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{\pi} Y_2^r & V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{\pi} Y_2^r + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{\pi} Y_2^r \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{\pi} Y_1^r + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{\pi} Y_1^r & V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{\pi} Y_1^r + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{\pi} Y_1^r & 0 & 0 & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{\pi} Y_1^r + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{\pi} Y_1^r & V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{\pi} Y_1^r + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{\pi} Y_1^r \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{\pi} Y_2^r + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{\pi} Y_2^r & V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{\pi} Y_2^r + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{\pi} Y_2^r & 0 & 0 & V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{\pi} Y_2^r + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{\pi} Y_2^r & V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{\pi} Y_2^r + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{\pi} Y_2^r \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{\pi} Y_1^r + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{\pi} Y_1^r & V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{\pi} Y_1^r + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{\pi} Y_1^r & 0 & 0 & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{\pi} Y_1^r + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{\pi} Y_1^r & V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{\pi} Y_1^r + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{\pi} Y_1^r \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{\pi} Y_2^r + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{\pi} Y_2^r & V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{\pi} Y_2^r + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{\pi} Y_2^r & 0 & 0 & V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{\pi} Y_2^r + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{\pi} Y_2^r & V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{\pi} Y_2^r + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{\pi} Y_2^r \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\text{rowSums}(\overline{\widehat{\text{VB}}[m:n, :]} * (\overline{\text{AmL}[3:4]})^T * \overline{\text{rowSums}(\text{Y})[3:4, :]}[m:n]) =$$

$$\left(\begin{array}{l} \text{V}_1^s \text{B}_{11}^{ss} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_2^s \text{B}_{21}^{ss} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_1^s \text{B}_{12}^{ss} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_2^s \text{B}_{22}^{ss} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{jl}^{rr} \text{Y}_1^r \\ \text{V}_1^s \text{B}_{11}^{ss} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^s \text{B}_{21}^{ss} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_1^s \text{B}_{12}^{ss} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^s \text{B}_{22}^{ss} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r \\ \text{V}_1^r \text{B}_{11}^{rs} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_2^r \text{B}_{21}^{rs} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_1^r \text{B}_{12}^{rs} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_2^r \text{B}_{22}^{rs} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{jl}^{rr} \text{Y}_1^r \\ \text{V}_1^r \text{B}_{11}^{rs} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^r \text{B}_{21}^{rs} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_1^r \text{B}_{12}^{rs} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^r \text{B}_{22}^{rs} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r \\ \text{V}_1^t \text{B}_{11}^{ts} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_2^t \text{B}_{21}^{ts} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_1^t \text{B}_{12}^{ts} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_2^t \text{B}_{22}^{ts} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{jl}^{rr} \text{Y}_1^r \\ \text{V}_1^t \text{B}_{11}^{ts} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^t \text{B}_{21}^{ts} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_1^t \text{B}_{12}^{ts} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^t \text{B}_{22}^{ts} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r \end{array} \right) \begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \left(\begin{array}{l} \text{V}_1^s \text{B}_{11}^{st} \sum_{j=1}^2 \text{A}_{1j}^{tr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_2^s \text{B}_{21}^{st} \sum_{j=1}^2 \text{A}_{1j}^{tr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_1^s \text{B}_{12}^{st} \sum_{j=1}^2 \text{A}_{2j}^{tr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_2^s \text{B}_{22}^{st} \sum_{j=1}^2 \text{A}_{2j}^{tr} \text{L}_{jl}^{rr} \text{Y}_1^r \\ \text{V}_1^s \text{B}_{11}^{st} \sum_{j=1}^2 \text{A}_{1j}^{tr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^s \text{B}_{21}^{st} \sum_{j=1}^2 \text{A}_{1j}^{tr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_1^s \text{B}_{12}^{st} \sum_{j=1}^2 \text{A}_{2j}^{tr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^s \text{B}_{22}^{st} \sum_{j=1}^2 \text{A}_{2j}^{tr} \text{L}_{j2}^{rr} \text{Y}_2^r \\ \text{V}_1^r \text{B}_{11}^{rt} \sum_{j=1}^2 \text{A}_{1j}^{tr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_2^r \text{B}_{21}^{rt} \sum_{j=1}^2 \text{A}_{1j}^{tr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_1^r \text{B}_{12}^{rt} \sum_{j=1}^2 \text{A}_{2j}^{tr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_2^r \text{B}_{22}^{rt} \sum_{j=1}^2 \text{A}_{2j}^{tr} \text{L}_{jl}^{rr} \text{Y}_1^r \\ \text{V}_1^r \text{B}_{11}^{rt} \sum_{j=1}^2 \text{A}_{1j}^{tr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^r \text{B}_{21}^{rt} \sum_{j=1}^2 \text{A}_{1j}^{tr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_1^r \text{B}_{12}^{rt} \sum_{j=1}^2 \text{A}_{2j}^{tr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^r \text{B}_{22}^{rt} \sum_{j=1}^2 \text{A}_{2j}^{tr} \text{L}_{j2}^{rr} \text{Y}_2^r \\ \text{V}_1^t \text{B}_{11}^{tt} \sum_{j=1}^2 \text{A}_{1j}^{tr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_2^t \text{B}_{21}^{tt} \sum_{j=1}^2 \text{A}_{1j}^{tr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_1^t \text{B}_{12}^{tt} \sum_{j=1}^2 \text{A}_{2j}^{tr} \text{L}_{jl}^{rr} \text{Y}_1^r + \text{V}_2^t \text{B}_{22}^{tt} \sum_{j=1}^2 \text{A}_{2j}^{tr} \text{L}_{jl}^{rr} \text{Y}_1^r \\ \text{V}_1^t \text{B}_{11}^{tt} \sum_{j=1}^2 \text{A}_{1j}^{tr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^t \text{B}_{21}^{tt} \sum_{j=1}^2 \text{A}_{1j}^{tr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_1^t \text{B}_{12}^{tt} \sum_{j=1}^2 \text{A}_{2j}^{tr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^t \text{B}_{22}^{tt} \sum_{j=1}^2 \text{A}_{2j}^{tr} \text{L}_{j2}^{rr} \text{Y}_2^r \end{array} \right)$$

此矩阵是将上一页的矩阵的第一列与第二列相加、第三列与第四列相加、第五列与第六列相加后得到的

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$$\text{rowSums}(\widehat{\text{VB}}[\text{m} : \text{n},]) * (\text{AmL}[3 : 4])^T * \text{rowSums}(\text{Y})[3 : 4,])[\text{m} : \text{n}][,1] =$$

$$\left(\begin{aligned} & \text{V}_1^s \text{B}_{11}^{ss} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j1}^{rr} \text{Y}_1^r + \text{V}_2^s \text{B}_{21}^{ss} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j1}^{rr} \text{Y}_1^r + \text{V}_1^s \text{B}_{12}^{ss} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j1}^{rr} \text{Y}_1^r + \text{V}_2^s \text{B}_{22}^{ss} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j1}^{rr} \text{Y}_1^r \\ & \text{V}_1^s \text{B}_{11}^{ss} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^s \text{B}_{21}^{ss} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_1^s \text{B}_{12}^{ss} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^s \text{B}_{22}^{ss} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r \\ & \text{V}_1^r \text{B}_{11}^{rs} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j1}^{rr} \text{Y}_1^r + \text{V}_2^r \text{B}_{21}^{rs} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j1}^{rr} \text{Y}_1^r + \text{V}_1^r \text{B}_{12}^{rs} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j1}^{rr} \text{Y}_1^r + \text{V}_2^r \text{B}_{22}^{rs} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j1}^{rr} \text{Y}_1^r \\ & \text{V}_1^r \text{B}_{11}^{rs} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^r \text{B}_{21}^{rs} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_1^r \text{B}_{12}^{rs} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^r \text{B}_{22}^{rs} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r \\ & \text{V}_1^t \text{B}_{11}^{ts} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j1}^{rr} \text{Y}_1^r + \text{V}_2^t \text{B}_{21}^{ts} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j1}^{rr} \text{Y}_1^r + \text{V}_1^t \text{B}_{12}^{ts} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j1}^{rr} \text{Y}_1^r + \text{V}_2^t \text{B}_{22}^{ts} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j1}^{rr} \text{Y}_1^r \\ & \text{V}_1^t \text{B}_{11}^{ts} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^t \text{B}_{21}^{ts} \sum_{j=1}^2 \text{A}_{1j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_1^t \text{B}_{12}^{ts} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r + \text{V}_2^t \text{B}_{22}^{ts} \sum_{j=1}^2 \text{A}_{2j}^{sr} \text{L}_{j2}^{rr} \text{Y}_2^r \end{aligned} \right)$$

$$\dim(\overline{\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[3:4])^T * \text{rowSums}(\text{Y})[3:4,])[m:n])[1])}) = c(N, G) =$$

$$\left(\begin{array}{cc} \mathbf{V}_1^s \mathbf{B}_{11}^{ss} \sum_{j=1}^2 \mathbf{A}_{1j}^{sr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_2^s \mathbf{B}_{21}^{ss} \sum_{j=1}^2 \mathbf{A}_{1j}^{sr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_1^s \mathbf{B}_{12}^{ss} \sum_{j=1}^2 \mathbf{A}_{2j}^{sr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_2^s \mathbf{B}_{22}^{ss} \sum_{j=1}^2 \mathbf{A}_{2j}^{sr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r & \mathbf{V}_1^s \mathbf{B}_{11}^{ss} \sum_{j=1}^2 \mathbf{A}_{1j}^{sr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_2^s \mathbf{B}_{21}^{ss} \sum_{j=1}^2 \mathbf{A}_{1j}^{sr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_1^s \mathbf{B}_{12}^{ss} \sum_{j=1}^2 \mathbf{A}_{2j}^{sr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_2^s \mathbf{B}_{22}^{ss} \sum_{j=1}^2 \mathbf{A}_{2j}^{sr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r \\ \mathbf{V}_1^r \mathbf{B}_{11}^{rs} \sum_{j=1}^2 \mathbf{A}_{1j}^{sr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_2^r \mathbf{B}_{21}^{rs} \sum_{j=1}^2 \mathbf{A}_{1j}^{sr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_1^r \mathbf{B}_{12}^{rs} \sum_{j=1}^2 \mathbf{A}_{2j}^{sr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_2^r \mathbf{B}_{22}^{rs} \sum_{j=1}^2 \mathbf{A}_{2j}^{sr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r & \mathbf{V}_1^r \mathbf{B}_{11}^{rs} \sum_{j=1}^2 \mathbf{A}_{1j}^{sr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_2^r \mathbf{B}_{21}^{rs} \sum_{j=1}^2 \mathbf{A}_{1j}^{sr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_1^r \mathbf{B}_{12}^{rs} \sum_{j=1}^2 \mathbf{A}_{2j}^{sr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_2^r \mathbf{B}_{22}^{rs} \sum_{j=1}^2 \mathbf{A}_{2j}^{sr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r \\ \mathbf{V}_1^t \mathbf{B}_{11}^{ts} \sum_{j=1}^2 \mathbf{A}_{1j}^{sr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_2^t \mathbf{B}_{21}^{ts} \sum_{j=1}^2 \mathbf{A}_{1j}^{sr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_1^t \mathbf{B}_{12}^{ts} \sum_{j=1}^2 \mathbf{A}_{2j}^{sr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_2^t \mathbf{B}_{22}^{ts} \sum_{j=1}^2 \mathbf{A}_{2j}^{sr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r & \mathbf{V}_1^t \mathbf{B}_{11}^{ts} \sum_{j=1}^2 \mathbf{A}_{1j}^{sr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_2^t \mathbf{B}_{21}^{ts} \sum_{j=1}^2 \mathbf{A}_{1j}^{sr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_1^t \mathbf{B}_{12}^{ts} \sum_{j=1}^2 \mathbf{A}_{2j}^{sr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_2^t \mathbf{B}_{22}^{ts} \sum_{j=1}^2 \mathbf{A}_{2j}^{sr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r \end{array} \right)^T$$

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$$\text{rowSums}(\text{dim}(\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[3:4])^T * \text{rowSums}(\text{Y})[3:4,])[m:n])[1]) = c(N, G)) =$$

$$\left(\begin{array}{cc} V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} Y_1^r + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} Y_1^r + V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r & V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} Y_2^r + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} Y_2^r + V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r \\ + V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} Y_1^r + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} Y_1^r + V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r & + V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} Y_2^r + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} Y_2^r + V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r \\ + V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} Y_1^r + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} Y_1^r + V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r & + V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} Y_2^r + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} Y_2^r + V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r \end{array} \right)^T$$

$$\text{rowSums}(\text{dim}(\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[3:4])^T * \text{rowSums}(\text{Y})[3:4,])[m:n])[1]) = c(N, G))$$

$$= \left(\sum_{k=s}^t V_1^k \sum_{j=1}^2 B_{1j}^{ks} \left(\sum_{i=1}^2 A_{ji}^{sr} L_{il}^{rr} \right) Y_1^r + \sum_{k=s}^t V_2^k \sum_{j=1}^2 B_{2j}^{ks} \left(\sum_{i=1}^2 A_{ji}^{sr} L_{il}^{rr} \right) Y_1^r \quad \sum_{k=s}^t V_1^k \sum_{j=1}^2 B_{1j}^{ks} \left(\sum_{i=1}^2 A_{ji}^{sr} L_{i2}^{rr} \right) Y_2^r + \sum_{k=s}^t V_2^k \sum_{j=1}^2 B_{2j}^{ks} \left(\sum_{i=1}^2 A_{ji}^{sr} L_{i2}^{rr} \right) Y_2^r \right)^T = \left(\sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{ks} \left(\sum_{i=1}^2 A_{ji}^{sr} L_{il}^{rr} \right) Y_1^r \quad \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{ks} \left(\sum_{i=1}^2 A_{ji}^{sr} L_{i2}^{rr} \right) Y_2^r \right)^T$$

含义说明:

r国从s国进口中间品并用于r国部门1参与GVC生产活动中包含的s国的增加值以及其他第三国的增加值

将r国从s国进口中间品的结果放入空矩阵

此矩阵的出口国为s国，直接进口国为r国，并且r国亦是目的地

$$\text{VBLAmY_back}^r =$$

$$\left(\sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{ks} \sum_{i=1}^2 A_{ji}^{sr} L_{il}^{rr} Y_1^r \quad \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{ks} \sum_{i=1}^2 A_{ji}^{sr} L_{i2}^{rr} Y_2^r \quad 0 \quad 0 \quad \text{ort}^{\text{tr}} \quad \text{export}^{\text{tr}} \right)^T$$

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rowSums((colSums($\widehat{VB}[m:n,]$) * ($\widehat{AmL}[3:4]$)^T * rowSums(Y)[3:4])[m:n]) =

$$\left(\begin{array}{l} V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} Y_1^r + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} Y_1^r + V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r \\ V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} Y_2^r + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} Y_2^r + V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} Y_1^r + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} Y_1^r + V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} Y_2^r + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} Y_2^r + V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} Y_1^r + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} Y_1^r + V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} Y_2^r + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} Y_2^r + V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r \end{array} \right) \begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \left(\begin{array}{l} V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} Y_1^r + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} Y_1^r + V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r \\ V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} Y_2^r + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} Y_2^r + V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r \\ V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} Y_1^r + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} Y_1^r + V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r \\ V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} Y_2^r + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} Y_2^r + V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r \\ V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} Y_1^r + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} Y_1^r + V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r \\ V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} Y_2^r + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} Y_2^r + V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r \end{array} \right)$$

$$\dim(\overline{\text{rowSums}(\overline{\text{colSums}(\widehat{\text{VB}}[m:n]) * (\text{AmL}[3:4])^T * \text{rowSums}(\text{Y})[3:4])}, [m:n])[3]) = c(N, G) =$$

$$\left(\begin{array}{cc} \mathbf{V}_1^s \mathbf{B}_{11}^{st} \sum_{j=1}^2 \mathbf{A}_{1j}^{tr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_2^s \mathbf{B}_{21}^{st} \sum_{j=1}^2 \mathbf{A}_{1j}^{tr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_1^s \mathbf{B}_{12}^{st} \sum_{j=1}^2 \mathbf{A}_{2j}^{tr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_2^s \mathbf{B}_{22}^{st} \sum_{j=1}^2 \mathbf{A}_{2j}^{tr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r & \mathbf{V}_1^s \mathbf{B}_{11}^{st} \sum_{j=1}^2 \mathbf{A}_{1j}^{tr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_2^s \mathbf{B}_{21}^{st} \sum_{j=1}^2 \mathbf{A}_{1j}^{tr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_1^s \mathbf{B}_{12}^{st} \sum_{j=1}^2 \mathbf{A}_{2j}^{tr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_2^s \mathbf{B}_{22}^{st} \sum_{j=1}^2 \mathbf{A}_{2j}^{tr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r \\ \mathbf{V}_1^r \mathbf{B}_{11}^{rt} \sum_{j=1}^2 \mathbf{A}_{1j}^{tr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_2^r \mathbf{B}_{21}^{rt} \sum_{j=1}^2 \mathbf{A}_{1j}^{tr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_1^r \mathbf{B}_{12}^{rt} \sum_{j=1}^2 \mathbf{A}_{2j}^{tr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_2^r \mathbf{B}_{22}^{rt} \sum_{j=1}^2 \mathbf{A}_{2j}^{tr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r & \mathbf{V}_1^r \mathbf{B}_{11}^{rt} \sum_{j=1}^2 \mathbf{A}_{1j}^{tr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_2^r \mathbf{B}_{21}^{rt} \sum_{j=1}^2 \mathbf{A}_{1j}^{tr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_1^r \mathbf{B}_{12}^{rt} \sum_{j=1}^2 \mathbf{A}_{2j}^{tr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_2^r \mathbf{B}_{22}^{rt} \sum_{j=1}^2 \mathbf{A}_{2j}^{tr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r \\ \mathbf{V}_1^t \mathbf{B}_{11}^{tt} \sum_{j=1}^2 \mathbf{A}_{1j}^{tr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_2^t \mathbf{B}_{21}^{tt} \sum_{j=1}^2 \mathbf{A}_{1j}^{tr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_1^t \mathbf{B}_{12}^{tt} \sum_{j=1}^2 \mathbf{A}_{2j}^{tr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r + \mathbf{V}_2^t \mathbf{B}_{22}^{tt} \sum_{j=1}^2 \mathbf{A}_{2j}^{tr} \mathbf{L}_{jl}^{rr} \mathbf{Y}_1^r & \mathbf{V}_1^t \mathbf{B}_{11}^{tt} \sum_{j=1}^2 \mathbf{A}_{1j}^{tr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_2^t \mathbf{B}_{21}^{tt} \sum_{j=1}^2 \mathbf{A}_{1j}^{tr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_1^t \mathbf{B}_{12}^{tt} \sum_{j=1}^2 \mathbf{A}_{2j}^{tr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r + \mathbf{V}_2^t \mathbf{B}_{22}^{tt} \sum_{j=1}^2 \mathbf{A}_{2j}^{tr} \mathbf{L}_{j2}^{rr} \mathbf{Y}_2^r \end{array} \right)^T$$

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$$\text{rowSums}(\text{dim}(\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[3:4])^T * \text{rowSums}(\text{Y})[3:4,])[m:n])[3]) = c(N, G)) =$$

$$\left(\begin{array}{cc} V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} Y_1^r + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} Y_1^r + V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r & V_1^s B_{11}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} Y_2^r + V_2^s B_{21}^{st} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} Y_2^r + V_1^s B_{12}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r + V_2^s B_{22}^{st} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r \\ + V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} Y_1^r + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} Y_1^r + V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r & + V_1^r B_{11}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} Y_2^r + V_2^r B_{21}^{rt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} Y_2^r + V_1^r B_{12}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r + V_2^r B_{22}^{rt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r \\ + V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} Y_1^r + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} Y_1^r + V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r & + V_1^t B_{11}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} Y_2^r + V_2^t B_{21}^{tt} \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} Y_2^r + V_1^t B_{12}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r + V_2^t B_{22}^{tt} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r \end{array} \right)^T$$

$$\text{rowSums}(\text{dim}(\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[3:4])^T * \text{rowSums}(\text{Y})[3:4,])[m:n])[3]) = c(N, G))$$

$$= \left(\sum_{k=s}^t V_1^k \sum_{j=1}^2 B_{1j}^{kt} \left(\sum_{i=1}^2 A_{ji}^{tr} L_{il}^{rr} \right) Y_1^r + \sum_{k=s}^t V_2^k \sum_{j=1}^2 B_{2j}^{kt} \left(\sum_{i=1}^2 A_{ji}^{tr} L_{il}^{rr} \right) Y_1^r \quad \sum_{k=s}^t V_1^k \sum_{j=1}^2 B_{1j}^{kt} \left(\sum_{i=1}^2 A_{ji}^{tr} L_{i2}^{rr} \right) Y_2^r + \sum_{k=s}^t V_2^k \sum_{j=1}^2 B_{2j}^{kt} \left(\sum_{i=1}^2 A_{ji}^{tr} L_{i2}^{rr} \right) Y_2^r \right)^T = \left(\sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kt} \left(\sum_{i=1}^2 A_{ji}^{tr} L_{il}^{rr} \right) Y_1^r \quad \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kt} \left(\sum_{i=1}^2 A_{ji}^{tr} L_{i2}^{rr} \right) Y_2^r \right)^T$$

含义说明：
r国从t国进口中间品并用于s国部门1参与GVC生产活动中包含的t国的增加值以及其他第三国的增加值
此矩阵的出口国为t国，直接进口国为r国，并且r国亦是目的地

将r国从t国进口中间品的结果放入空矩阵

$$\text{VBLAmY_back}^r =$$

$$\left(\begin{array}{cc} \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{ks} \left(\sum_{i=1}^2 A_{ji}^{sr} L_{il}^{rr} \right) Y_1^r & \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{ks} \left(\sum_{i=1}^2 A_{ji}^{sr} L_{i2}^{rr} \right) Y_2^r & 0 & 0 \\ \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kt} \left(\sum_{i=1}^2 A_{ji}^{tr} L_{il}^{rr} \right) Y_1^r & \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kt} \left(\sum_{i=1}^2 A_{ji}^{tr} L_{i2}^{rr} \right) Y_2^r \end{array} \right)^T$$

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$$L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix}$$

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$\hat{V}B = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} = \begin{pmatrix} V_1^s B_{11}^{ss} & V_1^s B_{12}^{ss} & V_1^s B_{11}^{sr} & V_1^s B_{12}^{sr} & V_1^s B_{11}^{st} & V_1^s B_{12}^{st} \\ V_2^s B_{21}^{ss} & V_2^s B_{22}^{ss} & V_2^s B_{21}^{sr} & V_2^s B_{22}^{sr} & V_2^s B_{21}^{st} & V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} & V_1^r B_{12}^{rs} & V_1^r B_{11}^{rr} & V_1^r B_{12}^{rr} & V_1^r B_{11}^{rt} & V_1^r B_{12}^{rt} \\ V_2^r B_{21}^{rs} & V_2^r B_{22}^{rs} & V_2^r B_{21}^{rr} & V_2^r B_{22}^{rr} & V_2^r B_{21}^{rt} & V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} & V_1^t B_{12}^{ts} & V_1^t B_{11}^{tr} & V_1^t B_{12}^{tr} & V_1^t B_{11}^{tt} & V_1^t B_{12}^{tt} \\ V_2^t B_{21}^{ts} & V_2^t B_{22}^{ts} & V_2^t B_{21}^{tr} & V_2^t B_{22}^{tr} & V_2^t B_{21}^{tt} & V_2^t B_{22}^{tt} \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix}$$

$$\text{rowSums}(Y)[5:6,] = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ Y_1^t \\ Y_2^t \end{pmatrix}$$

$$\overline{\text{rowSums}(Y)[5:6,]} = \begin{pmatrix} Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \\ Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \\ Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \end{pmatrix}$$

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$$AmL = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix} \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{sr} L_{j1}^{rr} & \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} & \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} \\ 0 & 0 & \sum_{j=1}^2 A_{2j}^{sr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} & \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} & \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & 0 & 0 & \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} & \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{tr} L_{j1}^{rr} & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & 0 & 0 \\ \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{tr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} & 0 & 0 \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\overline{\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[5:6])^T} =$$

$$\begin{pmatrix} \text{V}_1^s \text{B}_{11}^{ss} + \text{V}_2^s \text{B}_{21}^{ss} & \text{V}_1^s \text{B}_{12}^{ss} + \text{V}_2^s \text{B}_{22}^{ss} & \text{V}_1^s \text{B}_{11}^{sr} + \text{V}_2^s \text{B}_{21}^{sr} & \text{V}_1^s \text{B}_{12}^{sr} + \text{V}_2^s \text{B}_{22}^{sr} & \text{V}_1^s \text{B}_{11}^{st} + \text{V}_2^s \text{B}_{21}^{st} & \text{V}_1^s \text{B}_{12}^{st} + \text{V}_2^s \text{B}_{22}^{st} \\ \text{V}_1^r \text{B}_{11}^{rs} + \text{V}_2^r \text{B}_{21}^{rs} & \text{V}_1^r \text{B}_{12}^{rs} + \text{V}_2^r \text{B}_{22}^{rs} & \text{V}_1^r \text{B}_{11}^{rr} + \text{V}_2^r \text{B}_{21}^{rr} & \text{V}_1^r \text{B}_{12}^{rr} + \text{V}_2^r \text{B}_{22}^{rr} & \text{V}_1^r \text{B}_{11}^{rt} + \text{V}_2^r \text{B}_{21}^{rt} & \text{V}_1^r \text{B}_{12}^{rt} + \text{V}_2^r \text{B}_{22}^{rt} \\ \text{V}_1^t \text{B}_{11}^{ts} + \text{V}_2^t \text{B}_{21}^{ts} & \text{V}_1^t \text{B}_{12}^{ts} + \text{V}_2^t \text{B}_{22}^{ts} & \text{V}_1^t \text{B}_{11}^{tr} + \text{V}_2^t \text{B}_{21}^{tr} & \text{V}_1^t \text{B}_{12}^{tr} + \text{V}_2^t \text{B}_{22}^{tr} & \text{V}_1^t \text{B}_{11}^{tt} + \text{V}_2^t \text{B}_{21}^{tt} & \text{V}_1^t \text{B}_{12}^{tt} + \text{V}_2^t \text{B}_{22}^{tt} \end{pmatrix} * \begin{pmatrix} \sum_{j=1}^2 \text{A}_{1j}^{st} \text{L}_{j1}^{tt} & \sum_{j=1}^2 \text{A}_{2j}^{st} \text{L}_{j1}^{tt} & \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j1}^{tt} & \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j1}^{tt} & 0 & 0 \\ \sum_{j=1}^2 \text{A}_{1j}^{st} \text{L}_{j2}^{tt} & \sum_{j=1}^2 \text{A}_{2j}^{st} \text{L}_{j2}^{tt} & \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j2}^{tt} & \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j2}^{tt} & 0 & 0 \\ \sum_{j=1}^2 \text{A}_{1j}^{st} \text{L}_{j1}^{tt} & \sum_{j=1}^2 \text{A}_{2j}^{st} \text{L}_{j1}^{tt} & \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j1}^{tt} & \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j1}^{tt} & 0 & 0 \\ \sum_{j=1}^2 \text{A}_{1j}^{st} \text{L}_{j2}^{tt} & \sum_{j=1}^2 \text{A}_{2j}^{st} \text{L}_{j2}^{tt} & \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j2}^{tt} & \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j2}^{tt} & 0 & 0 \\ \sum_{j=1}^2 \text{A}_{1j}^{st} \text{L}_{j1}^{tt} & \sum_{j=1}^2 \text{A}_{2j}^{st} \text{L}_{j1}^{tt} & \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j1}^{tt} & \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j1}^{tt} & 0 & 0 \\ \sum_{j=1}^2 \text{A}_{1j}^{st} \text{L}_{j2}^{tt} & \sum_{j=1}^2 \text{A}_{2j}^{st} \text{L}_{j2}^{tt} & \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j2}^{tt} & \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j2}^{tt} & 0 & 0 \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[5:6])^T =$$

$$\begin{pmatrix} V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} & V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} & V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} & 0 & 0 \\ V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} & V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} & V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} & 0 & 0 \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} & V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} & V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} & 0 & 0 \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} & V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} & V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} & 0 & 0 \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} & V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} & V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} & 0 & 0 \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} & V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} & V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} & 0 & 0 \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\overline{\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[5:6])^T * \overline{\text{rowSums}(Y)[5:6,]}} =$$

$$\begin{pmatrix} V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} & V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} & V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} & 0 & 0 \\ V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} & V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} & V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} & 0 & 0 \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} & V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} & V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} & 0 & 0 \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} & V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} & V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} & 0 & 0 \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} & V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} & V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} & 0 & 0 \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} & V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} & V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} & 0 & 0 \end{pmatrix}$$

$$* \begin{pmatrix} Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \\ Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \\ Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[5:6])^T * \text{rowSums}(\text{Y})[5:6,] =$$

$$\begin{pmatrix} V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t & V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t & V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t & 0 & 0 \\ V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t & V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t & V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t & 0 & 0 \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t & V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t & V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t & 0 & 0 \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t & V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t & V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t & 0 & 0 \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t & V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t & V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t & 0 & 0 \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t & V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t & V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t & 0 & 0 \end{pmatrix}$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\text{rowSums}(\overline{(\text{colSums}(\widehat{\text{VB}}[m:n, :]) * (\text{AmL}[5:6])^T * \text{rowSums}(Y)[5:6, :])}, [m:n]) =$$

$$\left(\begin{array}{cc} V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t & 0 \\ V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t & 0 \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t & 0 \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t & 0 \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t & 0 \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t & 0 \end{array} \right)$$

此矩阵是将上一页的矩阵的第一列与第二列相加、第三列与第四列相加、第五列与第六列相加后得到的

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$$\text{rowSums}(\overline{(\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[5:6])^T * \text{rowSums}(\text{Y})[5:6,])}, [m:n])[1] = \begin{pmatrix} V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t \\ V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t \end{pmatrix}$$

$$\dim(\text{rowSums}(\overline{(\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[5:6])^T * \text{rowSums}(\text{Y})[5:6,])}, [m:n])[1]) = c(N, G) =$$

$$\begin{pmatrix} V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t \end{pmatrix}^T$$

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$$\text{rowSums}(\text{dim}(\text{rowSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[5:6])^T * \text{rowSums}(\text{Y})[5:6,])[m:n])[1]) = c(N, G) =$$

$$\begin{pmatrix} V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t \\ + V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & + V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t \\ + V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & + V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t \end{pmatrix}^T$$

$$\text{rowSums}(\text{dim}(\text{rowSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[5:6])^T * \text{rowSums}(\text{Y})[5:6,])[m:n])[1]) = c(N, G)$$

$$= \left(\sum_{k=s}^t V_1^k \sum_{j=1}^2 B_{1j}^{ks} \left(\sum_{i=1}^2 A_{ji}^{st} L_{il}^{tt} \right) Y_1^t + \sum_{k=s}^t V_2^k \sum_{j=1}^2 B_{2j}^{ks} \left(\sum_{i=1}^2 A_{ji}^{st} L_{il}^{tt} \right) Y_1^t \quad \sum_{k=s}^t V_1^k \sum_{j=1}^2 B_{1j}^{ks} \left(\sum_{i=1}^2 A_{ji}^{st} L_{i2}^{tt} \right) Y_2^t + \sum_{k=s}^t V_2^k \sum_{j=1}^2 B_{2j}^{ks} \left(\sum_{i=1}^2 A_{ji}^{st} L_{i2}^{tt} \right) Y_2^t \right)^T = \left(\sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{ks} \left(\sum_{i=1}^2 A_{ji}^{st} L_{il}^{tt} \right) Y_1^t \quad \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{ks} \left(\sum_{i=1}^2 A_{ji}^{st} L_{i2}^{tt} \right) Y_2^t \right)^T$$

含义说明:

t国从s国进口中间品并用于t国部门1参与GVC生产活动中包含的s国的增加值以及其他第三国的增加值

将t国从s国进口中间品的结果放入空矩阵

此矩阵的出口国为s国, 直接进口国为t国, 并且t国亦是目的地

$$\text{VBLAmY_back}^t =$$

$$\left(\sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{ks} \sum_{i=1}^2 A_{ji}^{st} L_{il}^{tt} Y_1^t \quad \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{ks} \sum_{i=1}^2 A_{ji}^{st} L_{i2}^{tt} Y_2^t \quad t^{rt} \quad \text{export}^{rt} \quad 0 \quad 0 \right)^T$$

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rowSums((colSums($\widehat{VB}[m:n,]$) * ($\widehat{AmL}[5:6]$)^T * rowSums(Y)[5:6,])[m:n]) =

$$\left(\begin{array}{cc} V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t & 0 \\ V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t & 0 \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t & 0 \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t & V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t & 0 \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{jl}^{tt} Y_1^t + V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{jl}^{tt} Y_1^t & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t & 0 \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} Y_2^t + V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} Y_2^t & V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t & 0 \end{array} \right)$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\text{rowSums}(\widehat{\text{VB}}[\text{m} : \text{n},]) * (\text{AmL}[:, 5 : 6])^T * \text{rowSums}(\text{Y})[5 : 6,])[\text{m} : \text{n}][, 2] =$$

$$\left(\begin{array}{l} \text{V}_1^s \text{B}_{11}^{sr} \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j1}^{tt} \text{Y}_1^t + \text{V}_2^s \text{B}_{21}^{sr} \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j1}^{tt} \text{Y}_1^t + \text{V}_1^s \text{B}_{12}^{sr} \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j1}^{tt} \text{Y}_1^t + \text{V}_2^s \text{B}_{22}^{sr} \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j1}^{tt} \text{Y}_1^t \\ \text{V}_1^s \text{B}_{11}^{sr} \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j2}^{tt} \text{Y}_2^t + \text{V}_2^s \text{B}_{21}^{sr} \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j2}^{tt} \text{Y}_2^t + \text{V}_1^s \text{B}_{12}^{sr} \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j2}^{tt} \text{Y}_2^t + \text{V}_2^s \text{B}_{22}^{sr} \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j2}^{tt} \text{Y}_2^t \\ \text{V}_1^r \text{B}_{11}^{rr} \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j1}^{tt} \text{Y}_1^t + \text{V}_2^r \text{B}_{21}^{rr} \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j1}^{tt} \text{Y}_1^t + \text{V}_1^r \text{B}_{12}^{rr} \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j1}^{tt} \text{Y}_1^t + \text{V}_2^r \text{B}_{22}^{rr} \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j1}^{tt} \text{Y}_1^t \\ \text{V}_1^r \text{B}_{11}^{rr} \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j2}^{tt} \text{Y}_2^t + \text{V}_2^r \text{B}_{21}^{rr} \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j2}^{tt} \text{Y}_2^t + \text{V}_1^r \text{B}_{12}^{rr} \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j2}^{tt} \text{Y}_2^t + \text{V}_2^r \text{B}_{22}^{rr} \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j2}^{tt} \text{Y}_2^t \\ \text{V}_1^t \text{B}_{11}^{tr} \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j1}^{tt} \text{Y}_1^t + \text{V}_2^t \text{B}_{21}^{tr} \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j1}^{tt} \text{Y}_1^t + \text{V}_1^t \text{B}_{12}^{tr} \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j1}^{tt} \text{Y}_1^t + \text{V}_2^t \text{B}_{22}^{tr} \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j1}^{tt} \text{Y}_1^t \\ \text{V}_1^t \text{B}_{11}^{tr} \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j2}^{tt} \text{Y}_2^t + \text{V}_2^t \text{B}_{21}^{tr} \sum_{j=1}^2 \text{A}_{1j}^{rt} \text{L}_{j2}^{tt} \text{Y}_2^t + \text{V}_1^t \text{B}_{12}^{tr} \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j2}^{tt} \text{Y}_2^t + \text{V}_2^t \text{B}_{22}^{tr} \sum_{j=1}^2 \text{A}_{2j}^{rt} \text{L}_{j2}^{tt} \text{Y}_2^t \end{array} \right)$$

$$\dim(\overline{\text{rowSums}((\widehat{\text{VB}}[\text{m}:\text{n},]) * (\text{AmL}[,5:6])^T * \overline{\text{rowSums}(\text{Y}[5:6,])[\text{m}:\text{n}])[,2]})} = \text{c}(\text{N}, \text{G}) =$$

$$\left(\begin{array}{cc} \mathbf{V}_1^s \mathbf{B}_{11}^{sr} \sum_{j=1}^2 \mathbf{A}_{1j}^r \mathbf{L}_{jl}^t \mathbf{Y}_1^t + \mathbf{V}_2^s \mathbf{B}_{21}^{sr} \sum_{j=1}^2 \mathbf{A}_{1j}^r \mathbf{L}_{jl}^t \mathbf{Y}_1^t + \mathbf{V}_1^s \mathbf{B}_{12}^{sr} \sum_{j=1}^2 \mathbf{A}_{2j}^r \mathbf{L}_{jl}^t \mathbf{Y}_1^t + \mathbf{V}_2^s \mathbf{B}_{22}^{sr} \sum_{j=1}^2 \mathbf{A}_{2j}^r \mathbf{L}_{jl}^t \mathbf{Y}_1^t & \mathbf{V}_1^s \mathbf{B}_{11}^{sr} \sum_{j=1}^2 \mathbf{A}_{1j}^r \mathbf{L}_{j2}^t \mathbf{Y}_2^t + \mathbf{V}_2^s \mathbf{B}_{21}^{sr} \sum_{j=1}^2 \mathbf{A}_{1j}^r \mathbf{L}_{j2}^t \mathbf{Y}_2^t + \mathbf{V}_1^s \mathbf{B}_{12}^{sr} \sum_{j=1}^2 \mathbf{A}_{2j}^r \mathbf{L}_{j2}^t \mathbf{Y}_2^t + \mathbf{V}_2^s \mathbf{B}_{22}^{sr} \sum_{j=1}^2 \mathbf{A}_{2j}^r \mathbf{L}_{j2}^t \mathbf{Y}_2^t \\ \mathbf{V}_1^r \mathbf{B}_{11}^{rr} \sum_{j=1}^2 \mathbf{A}_{1j}^r \mathbf{L}_{jl}^t \mathbf{Y}_1^t + \mathbf{V}_2^r \mathbf{B}_{21}^{rr} \sum_{j=1}^2 \mathbf{A}_{1j}^r \mathbf{L}_{jl}^t \mathbf{Y}_1^t + \mathbf{V}_1^r \mathbf{B}_{12}^{rr} \sum_{j=1}^2 \mathbf{A}_{2j}^r \mathbf{L}_{jl}^t \mathbf{Y}_1^t + \mathbf{V}_2^r \mathbf{B}_{22}^{rr} \sum_{j=1}^2 \mathbf{A}_{2j}^r \mathbf{L}_{jl}^t \mathbf{Y}_1^t & \mathbf{V}_1^r \mathbf{B}_{11}^{rr} \sum_{j=1}^2 \mathbf{A}_{1j}^r \mathbf{L}_{j2}^t \mathbf{Y}_2^t + \mathbf{V}_2^r \mathbf{B}_{21}^{rr} \sum_{j=1}^2 \mathbf{A}_{1j}^r \mathbf{L}_{j2}^t \mathbf{Y}_2^t + \mathbf{V}_1^r \mathbf{B}_{12}^{rr} \sum_{j=1}^2 \mathbf{A}_{2j}^r \mathbf{L}_{j2}^t \mathbf{Y}_2^t + \mathbf{V}_2^r \mathbf{B}_{22}^{rr} \sum_{j=1}^2 \mathbf{A}_{2j}^r \mathbf{L}_{j2}^t \mathbf{Y}_2^t \\ \mathbf{V}_1^t \mathbf{B}_{11}^{tr} \sum_{j=1}^2 \mathbf{A}_{1j}^r \mathbf{L}_{jl}^t \mathbf{Y}_1^t + \mathbf{V}_2^t \mathbf{B}_{21}^{tr} \sum_{j=1}^2 \mathbf{A}_{1j}^r \mathbf{L}_{jl}^t \mathbf{Y}_1^t + \mathbf{V}_1^t \mathbf{B}_{12}^{tr} \sum_{j=1}^2 \mathbf{A}_{2j}^r \mathbf{L}_{jl}^t \mathbf{Y}_1^t + \mathbf{V}_2^t \mathbf{B}_{22}^{tr} \sum_{j=1}^2 \mathbf{A}_{2j}^r \mathbf{L}_{jl}^t \mathbf{Y}_1^t & \mathbf{V}_1^t \mathbf{B}_{11}^{tr} \sum_{j=1}^2 \mathbf{A}_{1j}^r \mathbf{L}_{j2}^t \mathbf{Y}_2^t + \mathbf{V}_2^t \mathbf{B}_{21}^{tr} \sum_{j=1}^2 \mathbf{A}_{1j}^r \mathbf{L}_{j2}^t \mathbf{Y}_2^t + \mathbf{V}_1^t \mathbf{B}_{12}^{tr} \sum_{j=1}^2 \mathbf{A}_{2j}^r \mathbf{L}_{j2}^t \mathbf{Y}_2^t + \mathbf{V}_2^t \mathbf{B}_{22}^{tr} \sum_{j=1}^2 \mathbf{A}_{2j}^r \mathbf{L}_{j2}^t \mathbf{Y}_2^t \end{array} \right)^T$$

FGY_GVC分解至国家_行业.国家——V(B-L)Y分解

$$\text{rowSums}(\text{dim}(\text{rowSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[5:6])^T * \text{rowSums}(Y)[5:6,])[m:n])[2]) = c(N, G) =$$

$$\begin{pmatrix} V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t & V_1^s B_{11}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_2^s B_{21}^{sr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_1^s B_{12}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t + V_2^s B_{22}^{sr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t \\ + V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t & + V_1^r B_{11}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_2^r B_{21}^{rr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_1^r B_{12}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t + V_2^r B_{22}^{rr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t \\ + V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{jl}^{tt} Y_1^t + V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{jl}^{tt} Y_1^t & + V_1^t B_{11}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_2^t B_{21}^{tr} \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} Y_2^t + V_1^t B_{12}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t + V_2^t B_{22}^{tr} \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} Y_2^t \end{pmatrix}^T$$

$$\text{rowSums}(\text{dim}(\text{rowSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmL}[5:6])^T * \text{rowSums}(Y)[5:6,])[m:n])[2]) = c(N, G))$$

$$= \left(\sum_{k=s}^t V_1^k \sum_{j=1}^2 B_{1j}^{kr} \left(\sum_{i=1}^2 A_{ji}^{rt} L_{il}^{tt} \right) Y_1^t + \sum_{k=s}^t V_2^k \sum_{j=1}^2 B_{2j}^{kr} \left(\sum_{i=1}^2 A_{ji}^{rt} L_{il}^{tt} \right) Y_1^t \quad \sum_{k=s}^t V_1^k \sum_{j=1}^2 B_{1j}^{kr} \left(\sum_{i=1}^2 A_{ji}^{rt} L_{i2}^{tt} \right) Y_2^t + \sum_{k=s}^t V_2^k \sum_{j=1}^2 B_{2j}^{kr} \left(\sum_{i=1}^2 A_{ji}^{rt} L_{i2}^{tt} \right) Y_2^t \right)^T = \left(\sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kr} \left(\sum_{i=1}^2 A_{ji}^{rt} L_{il}^{tt} \right) Y_1^t \quad \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kr} \left(\sum_{i=1}^2 A_{ji}^{rt} L_{i2}^{tt} \right) Y_2^t \right)^T$$

含义说明：

t国从r国进口中间品并用于t国部门1参与GVC生产活动中包含的r国的增加值以及其他第三国的增加值
此矩阵的出口国为r国，直接进口国为t国，并且t国亦是目的地

将t国从r国进口中间品的结果放入空矩阵

$$\text{VBLAmY_back}^r =$$

$$\left(\sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{ks} \left(\sum_{i=1}^2 A_{ji}^{sr} L_{il}^{rr} \right) Y_1^r \quad \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{ks} \left(\sum_{i=1}^2 A_{ji}^{sr} L_{i2}^{rr} \right) Y_2^r \quad \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kr} \left(\sum_{i=1}^2 A_{ji}^{rt} L_{il}^{tt} \right) Y_1^t \quad \sum_{k=s, m=1}^{t, 2} V_m^k \sum_{j=1}^2 B_{mj}^{kr} \left(\sum_{i=1}^2 A_{ji}^{rt} L_{i2}^{tt} \right) Y_2^t \quad 0 \quad 0 \right)^T$$

九：Xyd_GVC(即PLy_GVC分子)分解至国家_行业.世界——对VBAmLLY分解

```
#GVC生产活动引致的国内总产出_世界  
#简单GVC引致的国内总产出  
Xyd_GVC_S<-colSums(VLL%*%Am%*%L%*%diag(rowSums(Yd)))
```

```
#复杂GVC引致的国内总产出  
z2 <- diag(VC)%*%B%*%Am  
zbm <- z2  
for (s in 1:G){  
  m <- 1+( s-1 )*N; n <- N+( s-1 )*N  
  zbm[ m:n,m:n ] <- 0  
}  
zbd <- z2 -zbm  
Xyd_GVC_D<-colSums(zbd)%*%L%*%L%*%diag(rowSums(Y))  
Xyd_GVC_FR<-(colSums(zbm))%*%L%*%L%*%diag(rowSums(Y))
```

```
Xyd_GVC_F<-Xyd_GVC_FR-Xyd_GVC_S
```

```
#GVC生产活动引致的国内总产出_世界  
Xyd_GVC_world<-colSums(diag(VC)%*%B%*%Am)%*%L%*%L%*%diag(rowSums(Y))  
或者也可以使用代码：  
Xyd_GVC_world_2<-colSums(diag(VC)%*%L%*%Am)%*%B%*%L%*%diag(rowSums(Y))
```

Xyd_GVC分解至国家_行业.世界——复杂+简单GVC引致的国内总产出

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} \quad L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} \quad \hat{V} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix} = \begin{pmatrix} \sum_{u=s}^t Y_1^{su} \\ \sum_{u=s}^t Y_2^{su} \\ \sum_{u=s}^t Y_1^{ru} \\ \sum_{u=s}^t Y_2^{ru} \\ \sum_{u=s}^t Y_1^{tu} \\ \sum_{u=s}^t Y_2^{tu} \end{pmatrix} \quad \widehat{\text{rowSums}(Y)} = \begin{pmatrix} Y_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_2^t \end{pmatrix} = \begin{pmatrix} \sum_{u=s}^t Y_1^{su} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sum_{u=s}^t Y_2^{su} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{u=s}^t Y_1^{ru} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sum_{u=s}^t Y_2^{ru} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{u=s}^t Y_1^{tu} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sum_{u=s}^t Y_2^{tu} \end{pmatrix}$$

Xyd_GVC分解至国家_行业.世界——复杂+简单GVC引致的国内总产出

$$LL = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix}$$

$$LL = \begin{pmatrix} \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \end{pmatrix}$$

$$Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$AmB = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix} \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

Xyd_GVC分解至国家_行业.世界——复杂+简单GVC引致的国内总产出

$$\widehat{VB} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} = \begin{pmatrix} V_1^s B_{11}^{ss} & V_1^s B_{12}^{ss} & V_1^s B_{11}^{sr} & V_1^s B_{12}^{sr} & V_1^s B_{11}^{st} & V_1^s B_{12}^{st} \\ V_2^s B_{21}^{ss} & V_2^s B_{22}^{ss} & V_2^s B_{21}^{sr} & V_2^s B_{22}^{sr} & V_2^s B_{21}^{st} & V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} & V_1^r B_{12}^{rs} & V_1^r B_{11}^{rr} & V_1^r B_{12}^{rr} & V_1^r B_{11}^{rt} & V_1^r B_{12}^{rt} \\ V_2^r B_{21}^{rs} & V_2^r B_{22}^{rs} & V_2^r B_{21}^{rr} & V_2^r B_{22}^{rr} & V_2^r B_{21}^{rt} & V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} & V_1^t B_{12}^{ts} & V_1^t B_{11}^{tr} & V_1^t B_{12}^{tr} & V_1^t B_{11}^{tt} & V_1^t B_{12}^{tt} \\ V_2^t B_{21}^{ts} & V_2^t B_{22}^{ts} & V_2^t B_{21}^{tr} & V_2^t B_{22}^{tr} & V_2^t B_{21}^{tt} & V_2^t B_{22}^{tt} \end{pmatrix}$$

$$\widehat{VBAm} = \begin{pmatrix} V_1^s B_{11}^{ss} & V_1^s B_{12}^{ss} & V_1^s B_{11}^{sr} & V_1^s B_{12}^{sr} & V_1^s B_{11}^{st} & V_1^s B_{12}^{st} \\ V_2^s B_{21}^{ss} & V_2^s B_{22}^{ss} & V_2^s B_{21}^{sr} & V_2^s B_{22}^{sr} & V_2^s B_{21}^{st} & V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} & V_1^r B_{12}^{rs} & V_1^r B_{11}^{rr} & V_1^r B_{12}^{rr} & V_1^r B_{11}^{rt} & V_1^r B_{12}^{rt} \\ V_2^r B_{21}^{rs} & V_2^r B_{22}^{rs} & V_2^r B_{21}^{rr} & V_2^r B_{22}^{rr} & V_2^r B_{21}^{rt} & V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} & V_1^t B_{12}^{ts} & V_1^t B_{11}^{tr} & V_1^t B_{12}^{tr} & V_1^t B_{11}^{tt} & V_1^t B_{12}^{tt} \\ V_2^t B_{21}^{ts} & V_2^t B_{22}^{ts} & V_2^t B_{21}^{tr} & V_2^t B_{22}^{tr} & V_2^t B_{21}^{tt} & V_2^t B_{22}^{tt} \end{pmatrix} \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$\widehat{VBAm} = \begin{pmatrix} \sum_{u \neq s, j=1}^{t,2} V_1^s B_{1j}^{su} A_{jl}^{us} & \sum_{u \neq s, j=1}^{t,2} V_1^s B_{1j}^{su} A_{j2}^{us} & \sum_{u \neq r, j=1}^{t,2} V_1^s B_{1j}^{su} A_{jl}^{ur} & \sum_{u \neq r, j=1}^{t,2} V_1^s B_{1j}^{su} A_{j2}^{ur} & \sum_{u \neq t, j=1}^{r,2} V_1^s B_{1j}^{su} A_{jl}^{ut} & \sum_{u \neq t, j=1}^{r,2} V_1^s B_{1j}^{su} A_{j2}^{ut} \\ \sum_{u \neq s, j=1}^{t,2} V_2^s B_{2j}^{su} A_{jl}^{us} & \sum_{u \neq s, j=1}^{t,2} V_2^s B_{2j}^{su} A_{j2}^{us} & \sum_{u \neq r, j=1}^{t,2} V_2^s B_{2j}^{su} A_{jl}^{ur} & \sum_{u \neq r, j=1}^{t,2} V_2^s B_{2j}^{su} A_{j2}^{ur} & \sum_{u \neq t, j=1}^{r,2} V_2^s B_{2j}^{su} A_{jl}^{ut} & \sum_{u \neq t, j=1}^{r,2} V_2^s B_{2j}^{su} A_{j2}^{ut} \\ \sum_{u \neq s, j=1}^{t,2} V_1^r B_{1j}^{ru} A_{jl}^{us} & \sum_{u \neq s, j=1}^{t,2} V_1^r B_{1j}^{ru} A_{j2}^{us} & \sum_{u \neq r, j=1}^{t,2} V_1^r B_{1j}^{ru} A_{jl}^{ur} & \sum_{u \neq r, j=1}^{t,2} V_1^r B_{1j}^{ru} A_{j2}^{ur} & \sum_{u \neq t, j=1}^{r,2} V_1^r B_{1j}^{ru} A_{jl}^{ut} & \sum_{u \neq t, j=1}^{r,2} V_1^r B_{1j}^{ru} A_{j2}^{ut} \\ \sum_{u \neq s, j=1}^{t,2} V_2^r B_{2j}^{ru} A_{jl}^{us} & \sum_{u \neq s, j=1}^{t,2} V_2^r B_{2j}^{ru} A_{j2}^{us} & \sum_{u \neq r, j=1}^{t,2} V_2^r B_{2j}^{ru} A_{jl}^{ur} & \sum_{u \neq r, j=1}^{t,2} V_2^r B_{2j}^{ru} A_{j2}^{ur} & \sum_{u \neq t, j=1}^{r,2} V_2^r B_{2j}^{ru} A_{jl}^{ut} & \sum_{u \neq t, j=1}^{r,2} V_2^r B_{2j}^{ru} A_{j2}^{ut} \\ \sum_{u \neq s, j=1}^{t,2} V_1^t B_{1j}^{tu} A_{jl}^{us} & \sum_{u \neq s, j=1}^{t,2} V_1^t B_{1j}^{tu} A_{j2}^{us} & \sum_{u \neq r, j=1}^{t,2} V_1^t B_{1j}^{tu} A_{jl}^{ur} & \sum_{u \neq r, j=1}^{t,2} V_1^t B_{1j}^{tu} A_{j2}^{ur} & \sum_{u \neq t, j=1}^{r,2} V_1^t B_{1j}^{tu} A_{jl}^{ut} & \sum_{u \neq t, j=1}^{r,2} V_1^t B_{1j}^{tu} A_{j2}^{ut} \\ \sum_{u \neq s, j=1}^{t,2} V_2^t B_{2j}^{tu} A_{jl}^{us} & \sum_{u \neq s, j=1}^{t,2} V_2^t B_{2j}^{tu} A_{j2}^{us} & \sum_{u \neq r, j=1}^{t,2} V_2^t B_{2j}^{tu} A_{jl}^{ur} & \sum_{u \neq r, j=1}^{t,2} V_2^t B_{2j}^{tu} A_{j2}^{ur} & \sum_{u \neq t, j=1}^{r,2} V_2^t B_{2j}^{tu} A_{jl}^{ut} & \sum_{u \neq t, j=1}^{r,2} V_2^t B_{2j}^{tu} A_{j2}^{ut} \end{pmatrix}$$

Xyd_GVC分解至国家_行业.世界——复杂+简单GVC引致的国内总产出

$$\widehat{VBAm} = \begin{pmatrix} V_1^s \sum_{u \neq s, j=1}^{t,2} B_{lj}^{su} A_{jl}^{us} & V_1^s \sum_{u \neq s, j=1}^{t,2} B_{lj}^{su} A_{j2}^{us} & V_1^s \sum_{u \neq r, j=1}^{t,2} B_{lj}^{su} A_{jl}^{ur} & V_1^s \sum_{u \neq r, j=1}^{t,2} B_{lj}^{su} A_{j2}^{ur} & V_1^s \sum_{u \neq t, j=1}^{r,2} B_{lj}^{su} A_{jl}^{ut} & V_1^s \sum_{u \neq t, j=1}^{r,2} B_{lj}^{su} A_{j2}^{ut} \\ V_2^s \sum_{u \neq s, j=1}^{t,2} B_{2j}^{su} A_{jl}^{us} & V_2^s \sum_{u \neq s, j=1}^{t,2} B_{2j}^{su} A_{j2}^{us} & V_2^s \sum_{u \neq r, j=1}^{t,2} B_{2j}^{su} A_{jl}^{ur} & V_2^s \sum_{u \neq r, j=1}^{t,2} B_{2j}^{su} A_{j2}^{ur} & V_2^s \sum_{u \neq t, j=1}^{r,2} B_{2j}^{su} A_{jl}^{ut} & V_2^s \sum_{u \neq t, j=1}^{r,2} B_{2j}^{su} A_{j2}^{ut} \\ V_1^r \sum_{u \neq s, j=1}^{t,2} B_{lj}^{ru} A_{jl}^{us} & V_1^r \sum_{u \neq s, j=1}^{t,2} B_{lj}^{ru} A_{j2}^{us} & V_1^r \sum_{u \neq r, j=1}^{t,2} B_{lj}^{ru} A_{jl}^{ur} & V_1^r \sum_{u \neq r, j=1}^{t,2} B_{lj}^{ru} A_{j2}^{ur} & V_1^r \sum_{u \neq t, j=1}^{r,2} B_{lj}^{ru} A_{jl}^{ut} & V_1^r \sum_{u \neq t, j=1}^{r,2} B_{lj}^{ru} A_{j2}^{ut} \\ V_2^r \sum_{u \neq s, j=1}^{t,2} B_{2j}^{ru} A_{jl}^{us} & V_2^r \sum_{u \neq s, j=1}^{t,2} B_{2j}^{ru} A_{j2}^{us} & V_2^r \sum_{u \neq r, j=1}^{t,2} B_{2j}^{ru} A_{jl}^{ur} & V_2^r \sum_{u \neq r, j=1}^{t,2} B_{2j}^{ru} A_{j2}^{ur} & V_2^r \sum_{u \neq t, j=1}^{r,2} B_{2j}^{ru} A_{jl}^{ut} & V_2^r \sum_{u \neq t, j=1}^{r,2} B_{2j}^{ru} A_{j2}^{ut} \\ V_1^t \sum_{u \neq s, j=1}^{t,2} B_{lj}^{tu} A_{jl}^{us} & V_1^t \sum_{u \neq s, j=1}^{t,2} B_{lj}^{tu} A_{j2}^{us} & V_1^t \sum_{u \neq r, j=1}^{t,2} B_{lj}^{tu} A_{jl}^{ur} & V_1^t \sum_{u \neq r, j=1}^{t,2} B_{lj}^{tu} A_{j2}^{ur} & V_1^t \sum_{u \neq t, j=1}^{r,2} B_{lj}^{tu} A_{jl}^{ut} & V_1^t \sum_{u \neq t, j=1}^{r,2} B_{lj}^{tu} A_{j2}^{ut} \\ V_2^t \sum_{u \neq s, j=1}^{t,2} B_{2j}^{tu} A_{jl}^{us} & V_2^t \sum_{u \neq s, j=1}^{t,2} B_{2j}^{tu} A_{j2}^{us} & V_2^t \sum_{u \neq r, j=1}^{t,2} B_{2j}^{tu} A_{jl}^{ur} & V_2^t \sum_{u \neq r, j=1}^{t,2} B_{2j}^{tu} A_{j2}^{ur} & V_2^t \sum_{u \neq t, j=1}^{r,2} B_{2j}^{tu} A_{jl}^{ut} & V_2^t \sum_{u \neq t, j=1}^{r,2} B_{2j}^{tu} A_{j2}^{ut} \end{pmatrix}$$

$$c \quad \widehat{\quad} = \left(\sum_{k=s, i=1}^{t,2} i^k \left(\sum_{u \neq s, j=1}^{t,2} k_{ij}^{ku} \quad_{jl}^{us} \right) \quad \sum_{k=s, i=1}^{t,2} i^k \left(\sum_{u \neq s, j=1}^{t,2} k_{ij}^{ku} \quad_{j2}^{us} \right) \quad \sum_{k=s, i=1}^{t,2} i^k \left(\sum_{u \neq r, j=1}^{t,2} k_{ij}^{ku} \quad_{jl}^{ur} \right) \quad \sum_{k=s, i=1}^{t,2} i^k \left(\sum_{u \neq r, j=1}^{t,2} k_{ij}^{ku} \quad_{j2}^{ur} \right) \quad \sum_{k=s, i=1}^{t,2} V_i^k \left(\sum_{u \neq t, j=1}^{r,2} B_{ij}^{ku} A_{jl}^{ut} \right) \quad \sum_{k=s, i=1}^{t,2} V_i^k \left(\sum_{u \neq t, j=1}^{r,2} B_{ij}^{ku} A_{j2}^{ut} \right) \right)$$

Xyd_GVC分解至国家_行业.世界——复杂+简单GVC引致的国内总产出

$$\widehat{\text{LLrowSums}(Y)} = \begin{pmatrix} \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \end{pmatrix} \begin{pmatrix} Y_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_2^t \end{pmatrix}$$

$$\widehat{\text{LLrowSums}(Y)} = \begin{pmatrix} \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} Y_1^s & \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} Y_2^s & 0 & 0 & 0 & 0 \\ \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} Y_1^s & \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} Y_1^r & \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} Y_2^r & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} Y_1^r & \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} Y_1^t & \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} Y_2^t \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} Y_1^t & \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} Y_2^t \end{pmatrix}$$

Xyd_GVC分解至国家_行业.世界——复杂+简单GVC引致的国内总产出

$$\text{colSums}(\widehat{\text{VBAm}}) \text{LLrowSums}(\text{Y}) =$$

$$\left(\begin{array}{cccccc} \sum_{k=s,i=1}^{t,2} V_i^k & \sum_{u \neq s,j=1}^{t,2} B_{ij}^{ku} A_{jl}^{us} & \sum_{k=s,i=1}^{t,2} V_i^k & \sum_{u \neq s,j=1}^{t,2} B_{ij}^{ku} A_{j2}^{us} & \sum_{k=s,i=1}^{t,2} V_i^k & \sum_{u \neq r,j=1}^{t,2} B_{ij}^{ku} A_{jl}^{ur} \\ \sum_{k=s,i=1}^{t,2} V_i^k & \sum_{u \neq r,j=1}^{t,2} B_{ij}^{ku} A_{j2}^{ur} & \sum_{k=s,i=1}^{t,2} V_i^k & \sum_{u \neq r,j=1}^{t,2} B_{ij}^{ku} A_{j2}^{ur} & \sum_{k=s,i=1}^{t,2} V_i^k & \sum_{u \neq t,j=1}^{r,2} B_{ij}^{ku} A_{jl}^{ut} \\ \sum_{k=s,i=1}^{t,2} V_i^k & \sum_{u \neq t,j=1}^{r,2} B_{ij}^{ku} A_{jl}^{ut} & \sum_{k=s,i=1}^{t,2} V_i^k & \sum_{u \neq t,j=1}^{r,2} B_{ij}^{ku} A_{j2}^{ut} & \sum_{k=s,i=1}^{t,2} V_i^k & \sum_{u \neq t,j=1}^{r,2} B_{ij}^{ku} A_{j2}^{ut} \end{array} \right)$$

$$\left(\begin{array}{cccccc} \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} Y_1^s & \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} Y_2^s & 0 & 0 & 0 & 0 \\ \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} Y_1^s & \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} Y_1^r & \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} Y_2^r & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} Y_1^r & \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} Y_1^t & \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} Y_2^t \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} Y_1^t & \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} Y_2^t \end{array} \right)$$

Xyd_GVC分解至国家_行业.世界——复杂+简单GVC引致的国内总产出

$$\text{colSums}(\widehat{\text{VBAm}})\widehat{\text{LLrowSums}}(\text{Y}) = \begin{pmatrix} \sum_{k=s,i=1}^{t,2} V_i^k \left(\sum_{u \neq s,j=1}^{t,2} B_{ij}^{ku} A_{jl}^{us} \right) \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} Y_1^s + \sum_{k=s,i=1}^{t,2} V_i^k \left(\sum_{u \neq s,j=1}^{t,2} B_{ij}^{ku} A_{j2}^{us} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} Y_1^s \\ \sum_{k=s,i=1}^{t,2} V_i^k \left(\sum_{u \neq s,j=1}^{t,2} B_{ij}^{ku} A_{jl}^{us} \right) \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} Y_2^s + \sum_{k=s,i=1}^{t,2} V_i^k \left(\sum_{u \neq s,j=1}^{t,2} B_{ij}^{ku} A_{j2}^{us} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s \\ \sum_{k=s,i=1}^{t,2} V_i^k \left(\sum_{u \neq r,j=1}^{t,2} B_{ij}^{ku} A_{jl}^{ur} \right) \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} Y_1^r + \sum_{k=s,i=1}^{t,2} V_i^k \left(\sum_{u \neq r,j=1}^{t,2} B_{ij}^{ku} A_{j2}^{ur} \right) \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} Y_1^r \\ \sum_{k=s,i=1}^{t,2} V_i^k \left(\sum_{u \neq r,j=1}^{t,2} B_{ij}^{ku} A_{jl}^{ur} \right) \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} Y_2^r + \sum_{k=s,i=1}^{t,2} V_i^k \left(\sum_{u \neq r,j=1}^{t,2} B_{ij}^{ku} A_{j2}^{ur} \right) \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} Y_2^r \\ \sum_{k=s,i=1}^{t,2} V_i^k \left(\sum_{u \neq t,j=1}^{r,2} B_{ij}^{ku} A_{jl}^{ut} \right) \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} Y_1^t + \sum_{k=s,i=1}^{t,2} V_i^k \left(\sum_{u \neq t,j=1}^{r,2} B_{ij}^{ku} A_{j2}^{ut} \right) \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} Y_1^t \\ \sum_{k=s,i=1}^{t,2} V_i^k \left(\sum_{u \neq t,j=1}^{r,2} B_{ij}^{ku} A_{jl}^{ut} \right) \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} Y_2^t + \sum_{k=s,i=1}^{t,2} V_i^k \left(\sum_{u \neq t,j=1}^{r,2} B_{ij}^{ku} A_{j2}^{ut} \right) \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} Y_2^t \end{pmatrix}^T$$

分解思路解析：

t国向s国出口中间品中包含的t国各部门的增加值的生产活动中引致的国内总产出+r国向s国出口中间品中包含的r国各部门的增加值的生产活动中引致的国内总产出

$$\text{colSums}(\widehat{\text{VBAm}})\widehat{\text{LLrowSums}}(\text{Y})[,2] = \begin{pmatrix} \sum_{k=s,i=1}^{t,2} V_i^k \left(\sum_{u \neq s,j=1}^{t,2} B_{ij}^{ku} A_{jl}^{us} \right) \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} Y_2^s + \sum_{k=s,i=1}^{t,2} V_i^k \left(\sum_{u \neq s,j=1}^{t,2} B_{ij}^{ku} A_{j2}^{us} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s \end{pmatrix}^T$$

以第二列为例：

u国各部门对s国出口中间品中包含的世界各部门的增加值被s国部门吸收，因此有：

$$\text{colSums}(\widehat{\text{VBAm}})\widehat{\text{LLrowSums}}(\text{Y})[,2] = \begin{pmatrix} V_1^s \left(\sum_{u \neq s,j=1}^{t,2} B_{1j}^{su} A_{jl}^{us} \right) \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} Y_2^s + V_1^s \left(\sum_{u \neq s,j=1}^{t,2} B_{1j}^{su} A_{j2}^{us} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s \\ + V_1^r \left(\sum_{u \neq s,j=1}^{t,2} B_{1j}^{ru} A_{jl}^{us} \right) \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} Y_2^s + V_1^r \left(\sum_{u \neq s,j=1}^{t,2} B_{1j}^{ru} A_{j2}^{us} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s \\ + V_1^t \left(\sum_{u \neq s,j=1}^{t,2} B_{1j}^{tu} A_{jl}^{us} \right) \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} Y_2^s + V_1^t \left(\sum_{u \neq s,j=1}^{t,2} B_{1j}^{tu} A_{j2}^{us} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s \end{pmatrix}$$

增加值来源国与部门：世界各国各部门

出口国部门：u国各部门

目的地：s国部门2

十：Xyd_GVC(即PLy_GVC分子)分解至国家_行业.国家——对VBAmLLY分解

VBAmLLY分解至国家_行业.国家的代码:

```
VB<-diag(VC)%*%B
VB_3<-array(0,dim=c(GN,GN))
for (ii in 1:G) {
  m=1+N*(ii-1) ; n=N+(ii-1)*N
  VB_1<-t(as.matrix(colSums(VB[m:n,])))
  VB_2<-matrix(VB_1,nrow=N,ncol=GN,byrow=TRUE)
  VB_3[m:n,1:GN]<-VB_2[1:N,]
}
AmLL<-Am%*%L%*%L
VBAmLL_2<-array(0,dim=c(GN,GN))
VBAmLLY_back<-array(0,dim=c(GN*G,1))
VBAmLLY_5<-array(0,dim=c(GN,1))
for (i in 1:G) {
  m=1+N*(i-1) ; n=N+(i-1)*N
  mm=1+GN*(i-1) ; nn=GN+(i-1)*GN
  AmLL_1<-t(array(AmLL[,m:n],dim=c(GN,GN)))
  VBAmLL<-VB_3*AmLL_1
  YY<-as.matrix(rowSums(Y))
  YY_1<-matrix(YY[m:n,],nrow=GN,ncol=GN,byrow = FALSE)
  VBAmLLY<-VBAmLL*YY_1
  for (jj in 1:G){
    ss=1+N*(jj-1) ; uu=N+(jj-1)*N
    VBAmLLY_2<-as.matrix(rowSums(VBAmLLY[,ss:uu]))
    VBAmLLY_3<-VBAmLLY_2
    dim(VBAmLLY_3)<-c(N,G)
    VBAmLLY_4<-as.matrix(rowSums(VBAmLLY_3))
    VBAmLLY_5[ss:uu,1]<-VBAmLLY_4[,1]
  }
  VBAmLLY_back[mm:nn,1]<-VBAmLLY_5[1:GN,1]
}
```

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$\text{colSums}(\widehat{\text{VBAm}})\text{LLrowSums}(\text{Y}) = \begin{pmatrix} 0 \\ 0 \\ \sum_{k=s,i=1}^{t,2} V_i^k (\sum_{j=1}^2 B_{ij}^{kr} A_{jl}^{rs}) \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} Y_1^s + \sum_{k=s,i=1}^{t,2} V_i^k (\sum_{j=1}^2 B_{ij}^{kr} A_{j2}^{rs}) \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} Y_1^s \\ \sum_{k=s,i=1}^{t,2} V_i^k (\sum_{j=1}^2 B_{ij}^{kr} A_{jl}^{rs}) \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} Y_2^s + \sum_{k=s,i=1}^{t,2} V_i^k (\sum_{j=1}^2 B_{ij}^{kr} A_{j2}^{rs}) \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s \\ \sum_{k=s,i=1}^{t,2} V_i^k (\sum_{j=1}^2 B_{ij}^{kt} A_{jl}^{ts}) \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} Y_1^s + \sum_{k=s,i=1}^{t,2} V_i^k (\sum_{j=1}^2 B_{ij}^{kt} A_{j2}^{ts}) \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} Y_1^s \\ \sum_{k=s,i=1}^{t,2} V_i^k (\sum_{j=1}^2 B_{ij}^{kt} A_{jl}^{ts}) \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} Y_2^s + \sum_{k=s,i=1}^{t,2} V_i^k (\sum_{j=1}^2 B_{ij}^{kt} A_{j2}^{ts}) \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s \end{pmatrix}^T = \begin{pmatrix} 0 \\ 0 \\ \sum_{k=s,i=1}^{t,2} V_i^k B_{i1}^{kr} \sum_{j=1}^2 A_{lj}^{rs} \sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} Y_1^s + \sum_{k=s,i=1}^{t,2} V_i^k B_{i2}^{kr} \sum_{j=1}^2 A_{2j}^{rs} \sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} Y_1^s \\ \sum_{k=s,i=1}^{t,2} V_i^k B_{i1}^{kr} \sum_{j=1}^2 A_{lj}^{rs} \sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} Y_2^s + \sum_{k=s,i=1}^{t,2} V_i^k B_{i2}^{kr} \sum_{j=1}^2 A_{2j}^{rs} \sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} Y_2^s \\ \sum_{k=s,i=1}^{t,2} V_i^k B_{i1}^{kt} \sum_{j=1}^2 A_{lj}^{ts} \sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} Y_1^s + \sum_{k=s,i=1}^{t,2} V_i^k B_{i2}^{kt} \sum_{j=1}^2 A_{2j}^{ts} \sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} Y_1^s \\ \sum_{k=s,i=1}^{t,2} V_i^k B_{i1}^{kt} \sum_{j=1}^2 A_{lj}^{ts} \sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} Y_2^s + \sum_{k=s,i=1}^{t,2} V_i^k B_{i2}^{kt} \sum_{j=1}^2 A_{2j}^{ts} \sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} Y_2^s \end{pmatrix}^T$$

$$\begin{aligned} \text{colSums}(\widehat{\text{VBAm}})\text{LLrowSums}(\text{Y})[3] &= \sum_{k=s,i=1}^{t,2} V_i^k (\sum_{j=1}^2 B_{ij}^{kr} A_{jl}^{rs}) \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} Y_1^s + \sum_{k=s,i=1}^{t,2} V_i^k (\sum_{j=1}^2 B_{ij}^{kr} A_{j2}^{rs}) \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} Y_1^s \\ &= \sum_{k=s,i=1}^{t,2} V_i^k B_{i1}^{kr} A_{11}^{rs} \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} Y_1^s + \sum_{k=s,i=1}^{t,2} V_i^k B_{i2}^{kr} A_{21}^{rs} \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} Y_1^s + \sum_{k=s,i=1}^{t,2} V_i^k B_{i1}^{kr} A_{12}^{rs} \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} Y_1^s + \sum_{k=s,i=1}^{t,2} V_i^k B_{i2}^{kr} A_{22}^{rs} \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} Y_1^s \\ &= \sum_{k=s,i=1}^{t,2} V_i^k B_{i1}^{kr} \sum_{j=1}^2 A_{lj}^{rs} \sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} Y_1^s + \sum_{k=s,i=1}^{t,2} V_i^k B_{i2}^{kr} \sum_{j=1}^2 A_{2j}^{rs} \sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} Y_1^s \end{aligned}$$

$$\begin{aligned} \text{colSums}(\widehat{\text{VBAm}})\text{LLrowSums}(\text{Y})[4] &= \sum_{k=s,i=1}^{t,2} V_i^k (\sum_{j=1}^2 B_{ij}^{kr} A_{jl}^{rs}) \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} Y_2^s + \sum_{k=s,i=1}^{t,2} V_i^k (\sum_{j=1}^2 B_{ij}^{kr} A_{j2}^{rs}) \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s \\ &= \sum_{k=s,i=1}^{t,2} V_i^k B_{i1}^{kr} A_{11}^{rs} \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} Y_2^s + \sum_{k=s,i=1}^{t,2} V_i^k B_{i2}^{kr} A_{21}^{rs} \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} Y_2^s + \sum_{k=s,i=1}^{t,2} V_i^k B_{i1}^{kr} A_{12}^{rs} \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s + \sum_{k=s,i=1}^{t,2} V_i^k B_{i2}^{kr} A_{22}^{rs} \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s \\ &= \sum_{k=s,i=1}^{t,2} V_i^k B_{i1}^{kr} \sum_{j=1}^2 A_{lj}^{rs} \sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} Y_2^s + \sum_{k=s,i=1}^{t,2} V_i^k B_{i2}^{kr} \sum_{j=1}^2 A_{2j}^{rs} \sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} Y_2^s \end{aligned}$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix}$$

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$\hat{VB} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} = \begin{pmatrix} V_1^s B_{11}^{ss} & V_1^s B_{12}^{ss} & V_1^s B_{11}^{sr} & V_1^s B_{12}^{sr} & V_1^s B_{11}^{st} & V_1^s B_{12}^{st} \\ V_2^s B_{21}^{ss} & V_2^s B_{22}^{ss} & V_2^s B_{21}^{sr} & V_2^s B_{22}^{sr} & V_2^s B_{21}^{st} & V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} & V_1^r B_{12}^{rs} & V_1^r B_{11}^{rr} & V_1^r B_{12}^{rr} & V_1^r B_{11}^{rt} & V_1^r B_{12}^{rt} \\ V_2^r B_{21}^{rs} & V_2^r B_{22}^{rs} & V_2^r B_{21}^{rr} & V_2^r B_{22}^{rr} & V_2^r B_{21}^{rt} & V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} & V_1^t B_{12}^{ts} & V_1^t B_{11}^{tr} & V_1^t B_{12}^{tr} & V_1^t B_{11}^{tt} & V_1^t B_{12}^{tt} \\ V_2^t B_{21}^{ts} & V_2^t B_{22}^{ts} & V_2^t B_{21}^{tr} & V_2^t B_{22}^{tr} & V_2^t B_{21}^{tt} & V_2^t B_{22}^{tt} \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix}$$

$$\text{rowSums}(Y)[1:2,] = \begin{pmatrix} Y_1^s \\ Y_2^s \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\overline{\text{rowSums}(Y)[1:2,]} = \begin{pmatrix} Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \end{pmatrix}$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$LL = \begin{pmatrix} \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \end{pmatrix}$$

$$AmLL = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix} \begin{pmatrix} \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \end{pmatrix}$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$AmLL = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & \sum_{j=1}^2 A_{1j}^{st} (\sum_{m=1}^2 L_{jm}^{tt} L_{m1}^{tt}) & \sum_{j=1}^2 A_{1j}^{st} (\sum_{m=1}^2 L_{jm}^{tt} L_{m2}^{tt}) \\ 0 & 0 & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{m=1}^2 L_{jm}^{tt} L_{m1}^{tt}) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{m=1}^2 L_{jm}^{tt} L_{m2}^{tt}) \\ \sum_{j=1}^2 A_{1j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{m=1}^2 L_{jm}^{tt} L_{m1}^{tt}) & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{m=1}^2 L_{jm}^{tt} L_{m2}^{tt}) \\ \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & 0 & 0 & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{m=1}^2 L_{jm}^{tt} L_{m1}^{tt}) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{m=1}^2 L_{jm}^{tt} L_{m2}^{tt}) \\ \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & 0 & 0 \\ \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & 0 & 0 \end{pmatrix}$$

$$\left(\begin{array}{cc} 0 & 0 \\ 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) \\ \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) \\ \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) \\ \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) \end{array} \right)$$

[illegible]

$$\overline{(\text{AmLL}[1:2])}^T = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) \end{pmatrix}$$

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$$\text{colSums}(\widehat{\text{VB}}[m:n,]) = \begin{pmatrix} V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \end{pmatrix}$$



将VB矩阵的第一行与第二行加总、第三
行与第四行加总、第五行与第六行加总

$$\overline{\text{colSums}(\widehat{\text{VB}}[m:n,])} = \begin{pmatrix} V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \end{pmatrix}$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$\overline{\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[1:2])^T} =$$

$$\left(\begin{array}{cccccc} V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \end{array} \right) * \left(\begin{array}{cccc} 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) & \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) & \sum_{j=1}^2 A_{1j}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) & \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) \end{array} \right)$$

[illegible]

$$\overline{\text{colSums}(\widehat{\mathbf{VB}}[\mathbf{m} : \mathbf{n},]) * (\mathbf{AmLL}[1:2])^T = \begin{pmatrix} 0 & 0 & \sum_{i=1}^2 V_i^s B_{il}^{sr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{ml}^{ss}) & \sum_{i=1}^2 V_i^s B_{i2}^{sr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{ml}^{ss}) & \sum_{i=1}^2 V_i^s B_{il}^{st} \sum_{j=1}^2 A_{lj}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{ml}^{ss}) & \sum_{i=1}^2 V_i^s B_{i2}^{st} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{ml}^{ss}) \\ 0 & 0 & \sum_{i=1}^2 V_i^s B_{il}^{sr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{i=1}^2 V_i^s B_{i2}^{sr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{i=1}^2 V_i^s B_{il}^{st} \sum_{j=1}^2 A_{lj}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{i=1}^2 V_i^s B_{i2}^{st} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) \\ 0 & 0 & \sum_{i=1}^2 V_i^r B_{il}^{rr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{ml}^{ss}) & \sum_{i=1}^2 V_i^r B_{i2}^{rr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{ml}^{ss}) & \sum_{i=1}^2 V_i^r B_{il}^{rt} \sum_{j=1}^2 A_{lj}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{ml}^{ss}) & \sum_{i=1}^2 V_i^r B_{i2}^{rt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{ml}^{ss}) \\ 0 & 0 & \sum_{i=1}^2 V_i^r B_{il}^{rr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{i=1}^2 V_i^r B_{i2}^{rr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{i=1}^2 V_i^r B_{il}^{rt} \sum_{j=1}^2 A_{lj}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{i=1}^2 V_i^r B_{i2}^{rt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) \\ 0 & 0 & \sum_{i=1}^2 V_i^t B_{il}^{tr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{ml}^{ss}) & \sum_{i=1}^2 V_i^t B_{i2}^{tr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{ml}^{ss}) & \sum_{i=1}^2 V_i^t B_{il}^{tt} \sum_{j=1}^2 A_{lj}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{ml}^{ss}) & \sum_{i=1}^2 V_i^t B_{i2}^{tt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{ml}^{ss}) \\ 0 & 0 & \sum_{i=1}^2 V_i^t B_{il}^{tr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{i=1}^2 V_i^t B_{i2}^{tr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{i=1}^2 V_i^t B_{il}^{tt} \sum_{j=1}^2 A_{lj}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{i=1}^2 V_i^t B_{i2}^{tt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) \end{pmatrix}$$

$$\overline{\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[1:2])^T * \text{rowSums}(\text{Y})[1:2,]} =$$

$$\overline{\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[1:2])^T * \text{rowSums}(\text{Y})[1:2,]} =$$

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$$\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[1:2])^T * \text{rowSums}(Y)[1:2,])[m:n]) =$$

$$\begin{pmatrix} 0 & \sum_{i=1}^2 V_i^s B_{il}^{sr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s + \sum_{i=1}^2 V_i^s B_{i2}^{sr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s & \sum_{i=1}^2 V_i^s B_{il}^{st} \sum_{j=1}^2 A_{lj}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s + \sum_{i=1}^2 V_i^s B_{i2}^{st} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s \\ 0 & \sum_{i=1}^2 V_i^s B_{il}^{sr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s + \sum_{i=1}^2 V_i^s B_{i2}^{sr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s & \sum_{i=1}^2 V_i^s B_{il}^{st} \sum_{j=1}^2 A_{lj}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s + \sum_{i=1}^2 V_i^s B_{i2}^{st} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s \\ 0 & \sum_{i=1}^2 V_i^r B_{il}^{rr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s + \sum_{i=1}^2 V_i^r B_{i2}^{rr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s & \sum_{i=1}^2 V_i^r B_{il}^{rt} \sum_{j=1}^2 A_{lj}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s + \sum_{i=1}^2 V_i^r B_{i2}^{rt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s \\ 0 & \sum_{i=1}^2 V_i^r B_{il}^{rr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s + \sum_{i=1}^2 V_i^r B_{i2}^{rr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s & \sum_{i=1}^2 V_i^r B_{il}^{rt} \sum_{j=1}^2 A_{lj}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s + \sum_{i=1}^2 V_i^r B_{i2}^{rt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s \\ 0 & \sum_{i=1}^2 V_i^t B_{il}^{tr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s + \sum_{i=1}^2 V_i^t B_{i2}^{tr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s & \sum_{i=1}^2 V_i^t B_{il}^{tt} \sum_{j=1}^2 A_{lj}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s + \sum_{i=1}^2 V_i^t B_{i2}^{tt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s \\ 0 & \sum_{i=1}^2 V_i^t B_{il}^{tr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s + \sum_{i=1}^2 V_i^t B_{i2}^{tr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s & \sum_{i=1}^2 V_i^t B_{il}^{tt} \sum_{j=1}^2 A_{lj}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s + \sum_{i=1}^2 V_i^t B_{i2}^{tt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s \end{pmatrix}$$

此矩阵是将上一页的矩阵的第一列与第二列相加、第三列与第四列相加、第五列与第六列相加后得到的

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$$\overline{\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[1:2])^T * \text{rowSums}(\text{Y})[1:2,])[m:n])[2]} = \begin{pmatrix} [\sum_{i=1}^2 V_i^s B_{il}^{sr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) + \sum_{i=1}^2 V_i^s B_{i2}^{sr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss})] Y_1^s \\ [\sum_{i=1}^2 V_i^s B_{il}^{sr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) + \sum_{i=1}^2 V_i^s B_{i2}^{sr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss})] Y_2^s \\ [\sum_{i=1}^2 V_i^r B_{il}^{rr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) + \sum_{i=1}^2 V_i^r B_{i2}^{rr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss})] Y_1^s \\ [\sum_{i=1}^2 V_i^r B_{il}^{rr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) + \sum_{i=1}^2 V_i^r B_{i2}^{rr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss})] Y_2^s \\ [\sum_{i=1}^2 V_i^t B_{il}^{tr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) + \sum_{i=1}^2 V_i^t B_{i2}^{tr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss})] Y_1^s \\ [\sum_{i=1}^2 V_i^t B_{il}^{tr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) + \sum_{i=1}^2 V_i^t B_{i2}^{tr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss})] Y_2^s \end{pmatrix}$$

$$\dim(\overline{\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[1:2])^T * \text{rowSums}(\text{Y})[1:2,])[m:n])[2]}) = c(N, G)$$

$$= \begin{pmatrix} [\sum_{i=1}^2 V_i^s B_{il}^{sr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) + \sum_{i=1}^2 V_i^s B_{i2}^{sr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss})] Y_1^s & [\sum_{i=1}^2 V_i^s B_{il}^{sr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) + \sum_{i=1}^2 V_i^s B_{i2}^{sr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss})] Y_2^s \\ [\sum_{i=1}^2 V_i^r B_{il}^{rr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) + \sum_{i=1}^2 V_i^r B_{i2}^{rr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss})] Y_1^s & [\sum_{i=1}^2 V_i^r B_{il}^{rr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) + \sum_{i=1}^2 V_i^r B_{i2}^{rr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss})] Y_2^s \\ [\sum_{i=1}^2 V_i^t B_{il}^{tr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) + \sum_{i=1}^2 V_i^t B_{i2}^{tr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss})] Y_1^s & [\sum_{i=1}^2 V_i^t B_{il}^{tr} \sum_{j=1}^2 A_{lj}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) + \sum_{i=1}^2 V_i^t B_{i2}^{tr} \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss})] Y_2^s \end{pmatrix}^T$$

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$$\text{rowSums}(\text{dim}(\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n, :]) * (\text{AmLL}[1:2])^T * \text{rowSums}(\text{Y})[1:2, :])[m:n])[2]) = c(N, G))$$

$$= \left(\left[\sum_{k=s, i=1}^{t, 2} V_i^k B_{il}^{kr} \sum_{j=1}^2 A_{lj}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) + \sum_{k=s, i=1}^{t, 2} V_i^k B_{i2}^{kr} \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) \right] Y_1^s \quad \left[\sum_{k=s, i=1}^{t, 2} V_i^k B_{il}^{kr} \sum_{j=1}^2 A_{lj}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) + \sum_{k=s, i=1}^{t, 2} V_i^k B_{i2}^{kr} \sum_{j=1}^2 A_{2j}^{rs} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) \right] Y_2^s \right)^T$$

$$= \left(\left[\sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{jl}^{rs} \right) \sum_{m=1}^2 L_{lm}^{ss} L_{m1}^{ss} + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{j2}^{rs} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} \right] Y_1^s \quad \left[\sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{jl}^{rs} \right) \sum_{m=1}^2 L_{lm}^{ss} L_{m2}^{ss} + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{j2}^{rs} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} \right] Y_2^s \right)^T$$

含义说明:

s国从r国进口中间品并用于s国部门1参与GVC生产活动中包含的r国的增加值以及其他第三国的增加值

此矩阵的出口国为r国, 直接进口国为s国, 并且s国亦是目的地

将s国从r国进口中间品的结果放入空矩阵

VBAmLLY_back^s =

$$\left(\begin{array}{cc} \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{jl}^{rs} \right) \sum_{m=1}^2 L_{lm}^{ss} L_{m1}^{ss} Y_1^s & \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{jl}^{rs} \right) \sum_{m=1}^2 L_{lm}^{ss} L_{m2}^{ss} Y_2^s \\ 0 & 0 \end{array} \quad \begin{array}{cc} \text{export}^{ts} & \text{export}^{ts} \end{array} \right)^T$$

$$+ \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{j2}^{rs} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} Y_1^s + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{j2}^{rs} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$\text{rowSums}(\overline{(\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[1:2])^T * \text{rowSums}(\text{Y})[1:2,])[m:n])}, [3]) = \begin{pmatrix} \sum_{i=1}^2 V_i^s B_{i1}^{st} \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s + \sum_{i=1}^2 V_i^s B_{i2}^{st} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s \\ \sum_{i=1}^2 V_i^s B_{i1}^{st} \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s + \sum_{i=1}^2 V_i^s B_{i2}^{st} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s \\ \sum_{i=1}^2 V_i^r B_{i1}^{rt} \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s + \sum_{i=1}^2 V_i^r B_{i2}^{rt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s \\ \sum_{i=1}^2 V_i^r B_{i1}^{rt} \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s + \sum_{i=1}^2 V_i^r B_{i2}^{rt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s \\ \sum_{i=1}^2 V_i^t B_{i1}^{tt} \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s + \sum_{i=1}^2 V_i^t B_{i2}^{tt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s \\ \sum_{i=1}^2 V_i^t B_{i1}^{tt} \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s + \sum_{i=1}^2 V_i^t B_{i2}^{tt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s \end{pmatrix}$$

$$\dim(\text{rowSums}(\overline{(\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[1:2])^T * \text{rowSums}(\text{Y})[1:2,])[m:n])}, [3]) = c(N, G)$$

$$= \begin{pmatrix} \sum_{i=1}^2 V_i^s B_{i1}^{st} \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s + \sum_{i=1}^2 V_i^s B_{i2}^{st} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s & \sum_{i=1}^2 V_i^s B_{i1}^{st} \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s + \sum_{i=1}^2 V_i^s B_{i2}^{st} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s \\ \sum_{i=1}^2 V_i^r B_{i1}^{rt} \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s + \sum_{i=1}^2 V_i^r B_{i2}^{rt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s & \sum_{i=1}^2 V_i^r B_{i1}^{rt} \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s + \sum_{i=1}^2 V_i^r B_{i2}^{rt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s \\ \sum_{i=1}^2 V_i^t B_{i1}^{tt} \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s + \sum_{i=1}^2 V_i^t B_{i2}^{tt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) Y_1^s & \sum_{i=1}^2 V_i^t B_{i1}^{tt} \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s + \sum_{i=1}^2 V_i^t B_{i2}^{tt} \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) Y_2^s \end{pmatrix}^T$$

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$$\text{rowSums}(\text{dim}(\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n, :]) * \overline{(\text{AmLL}[1:2])}^T * \overline{\text{rowSums}(\text{Y})[1:2, :])}, [m:n])[3]) = c(N, G))$$

$$= \left(\left[\sum_{k=s, i=1}^{t, 2} V_i^k B_{il}^{kt} \sum_{j=1}^2 A_{lj}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) + \sum_{k=s, i=1}^{t, 2} V_i^k B_{i2}^{kt} \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss} \right) \right] Y_1^s \quad \left[\sum_{k=s, i=1}^{t, 2} V_i^k B_{il}^{kt} \sum_{j=1}^2 A_{lj}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) + \sum_{k=s, i=1}^{t, 2} V_i^k B_{i2}^{kt} \sum_{j=1}^2 A_{2j}^{ts} \left(\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss} \right) \right] Y_2^s \right)^T$$

$$= \left(\left[\sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kt} A_{jl}^{ts} \right) \sum_{m=1}^2 L_{lm}^{ss} L_{m1}^{ss} + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kt} A_{j2}^{ts} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} \right] Y_1^s \quad \left[\sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kt} A_{jl}^{ts} \right) \sum_{m=1}^2 L_{lm}^{ss} L_{m2}^{ss} + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kt} A_{j2}^{ts} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} \right] Y_2^s \right)^T$$

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$$\begin{pmatrix} 0 & 0 & \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{jl}^{rs} \right) \sum_{m=1}^2 L_{lm}^{ss} L_{m1}^{ss} Y_1^s & \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{j2}^{rs} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} Y_1^s & \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{jl}^{ts} \right) \sum_{m=1}^2 L_{lm}^{ss} L_{m1}^{ss} Y_1^s & \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{j2}^{ts} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} Y_1^s \\ + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{j2}^{rs} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} Y_1^s & + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{j2}^{rs} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s & + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{j2}^{ts} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} Y_1^s & + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kr} A_{j2}^{ts} \right) \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} Y_2^s \end{pmatrix}^T$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix}$$

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$\hat{VB} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} = \begin{pmatrix} V_1^s B_{11}^{ss} & V_1^s B_{12}^{ss} & V_1^s B_{11}^{sr} & V_1^s B_{12}^{sr} & V_1^s B_{11}^{st} & V_1^s B_{12}^{st} \\ V_2^s B_{21}^{ss} & V_2^s B_{22}^{ss} & V_2^s B_{21}^{sr} & V_2^s B_{22}^{sr} & V_2^s B_{21}^{st} & V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} & V_1^r B_{12}^{rs} & V_1^r B_{11}^{rr} & V_1^r B_{12}^{rr} & V_1^r B_{11}^{rt} & V_1^r B_{12}^{rt} \\ V_2^r B_{21}^{rs} & V_2^r B_{22}^{rs} & V_2^r B_{21}^{rr} & V_2^r B_{22}^{rr} & V_2^r B_{21}^{rt} & V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} & V_1^t B_{12}^{ts} & V_1^t B_{11}^{tr} & V_1^t B_{12}^{tr} & V_1^t B_{11}^{tt} & V_1^t B_{12}^{tt} \\ V_2^t B_{21}^{ts} & V_2^t B_{22}^{ts} & V_2^t B_{21}^{tr} & V_2^t B_{22}^{tr} & V_2^t B_{21}^{tt} & V_2^t B_{22}^{tt} \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix}$$

$$\text{rowSums}(Y)[3:4,] = \begin{pmatrix} 0 \\ 0 \\ Y_1^r \\ Y_2^r \\ 0 \\ 0 \end{pmatrix}$$

$$\overline{\text{rowSums}(Y)[3:4,]} = \begin{pmatrix} Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \end{pmatrix}$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$LL = \begin{pmatrix} \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \end{pmatrix}$$

$$AmLL = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix} \begin{pmatrix} \sum_{m=1}^2 L_{1m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{1m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ \sum_{m=1}^2 L_{2m}^{ss} L_{m1}^{ss} & \sum_{m=1}^2 L_{2m}^{ss} L_{m2}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{1m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{1m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} & \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{1m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{1m}^{tt} L_{m2}^{tt} \\ 0 & 0 & 0 & 0 & \sum_{m=1}^2 L_{2m}^{tt} L_{m1}^{tt} & \sum_{m=1}^2 L_{2m}^{tt} L_{m2}^{tt} \end{pmatrix}$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$AmLL = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & \sum_{j=1}^2 A_{1j}^{st} (\sum_{m=1}^2 L_{jm}^{tt} L_{m1}^{tt}) & \sum_{j=1}^2 A_{1j}^{st} (\sum_{m=1}^2 L_{jm}^{tt} L_{m2}^{tt}) \\ 0 & 0 & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{m=1}^2 L_{jm}^{tt} L_{m1}^{tt}) & \sum_{j=1}^2 A_{2j}^{st} (\sum_{m=1}^2 L_{jm}^{tt} L_{m2}^{tt}) \\ \sum_{j=1}^2 A_{1j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{1j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{m=1}^2 L_{jm}^{tt} L_{m1}^{tt}) & \sum_{j=1}^2 A_{1j}^{rt} (\sum_{m=1}^2 L_{jm}^{tt} L_{m2}^{tt}) \\ \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{2j}^{rs} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & 0 & 0 & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{m=1}^2 L_{jm}^{tt} L_{m1}^{tt}) & \sum_{j=1}^2 A_{2j}^{rt} (\sum_{m=1}^2 L_{jm}^{tt} L_{m2}^{tt}) \\ \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{1j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & 0 & 0 \\ \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m1}^{ss}) & \sum_{j=1}^2 A_{2j}^{ts} (\sum_{m=1}^2 L_{jm}^{ss} L_{m2}^{ss}) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & 0 & 0 \end{pmatrix}$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$\text{colSums}(\hat{\text{VB}}[m:n,]) = \begin{pmatrix} V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \end{pmatrix}$$



将VB矩阵的第一行与第二行加总、第三
行与第四行加总、第五行与第六行加总

$$\overline{\text{colSums}(\hat{\text{VB}}[m:n,])} = \begin{pmatrix} V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \end{pmatrix}$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$\overline{\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[3:4])^T} =$$

$$\begin{pmatrix} V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^s B_{11}^{ss} + V_2^s B_{21}^{ss} & V_1^s B_{12}^{ss} + V_2^s B_{22}^{ss} & V_1^s B_{11}^{sr} + V_2^s B_{21}^{sr} & V_1^s B_{12}^{sr} + V_2^s B_{22}^{sr} & V_1^s B_{11}^{st} + V_2^s B_{21}^{st} & V_1^s B_{12}^{st} + V_2^s B_{22}^{st} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^r B_{11}^{rs} + V_2^r B_{21}^{rs} & V_1^r B_{12}^{rs} + V_2^r B_{22}^{rs} & V_1^r B_{11}^{rr} + V_2^r B_{21}^{rr} & V_1^r B_{12}^{rr} + V_2^r B_{22}^{rr} & V_1^r B_{11}^{rt} + V_2^r B_{21}^{rt} & V_1^r B_{12}^{rt} + V_2^r B_{22}^{rt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \\ V_1^t B_{11}^{ts} + V_2^t B_{21}^{ts} & V_1^t B_{12}^{ts} + V_2^t B_{22}^{ts} & V_1^t B_{11}^{tr} + V_2^t B_{21}^{tr} & V_1^t B_{12}^{tr} + V_2^t B_{22}^{tr} & V_1^t B_{11}^{tt} + V_2^t B_{21}^{tt} & V_1^t B_{12}^{tt} + V_2^t B_{22}^{tt} \end{pmatrix} * \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) \\ \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) & \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) \end{pmatrix}$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[3:4])^T =$$

$$\left(\begin{array}{cccc} V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) & V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) & 0 & 0 \\ V_1^s B_{11}^{ss} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) + V_2^s B_{21}^{ss} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) & V_1^s B_{12}^{ss} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) + V_2^s B_{22}^{ss} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) & 0 & 0 \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) & V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) & 0 & 0 \\ V_1^r B_{11}^{rs} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) + V_2^r B_{21}^{rs} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) & V_1^r B_{12}^{rs} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) + V_2^r B_{22}^{rs} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) & 0 & 0 \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) & V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) & 0 & 0 \\ V_1^t B_{11}^{ts} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) + V_2^t B_{21}^{ts} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) & V_1^t B_{12}^{ts} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) + V_2^t B_{22}^{ts} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) & 0 & 0 \end{array} \right)$$

$$\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[3:4])^T = \left(\begin{array}{cccc} \sum_{i=1}^2 V_i^s B_{i1}^{ss} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) & \sum_{i=1}^2 V_i^s B_{i2}^{ss} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) & 0 & 0 \\ \sum_{i=1}^2 V_i^s B_{i1}^{ss} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) & \sum_{i=1}^2 V_i^s B_{i2}^{ss} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) & 0 & 0 \\ \sum_{i=1}^2 V_i^r B_{i1}^{rs} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) & \sum_{i=1}^2 V_i^r B_{i2}^{rs} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) & 0 & 0 \\ \sum_{i=1}^2 V_i^r B_{i1}^{rs} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) & \sum_{i=1}^2 V_i^r B_{i2}^{rs} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) & 0 & 0 \\ \sum_{i=1}^2 V_i^t B_{i1}^{ts} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) & \sum_{i=1}^2 V_i^t B_{i2}^{ts} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m1}^r) & 0 & 0 \\ \sum_{i=1}^2 V_i^t B_{i1}^{ts} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) & \sum_{i=1}^2 V_i^t B_{i2}^{ts} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^r L_{m2}^r) & 0 & 0 \end{array} \right)$$

$$\overline{\text{colSums}(\widehat{\text{VB}}[m:n,])} * \overline{(\text{AmLL}[3:4])}^T * \overline{\text{rowSums}(\text{Y})[3:4,]} =$$

$$\overline{\text{colSums}(\hat{\text{VB}}[\text{m:n},]) * (\text{AmLL}[3:4])^T * \text{rowSums}(\text{Y})[3:4,]} =$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[3:4])^T * \text{rowSums}(\text{Y})[3:4,])[m:n]) =$$

$$\left(\begin{array}{cc} \sum_{i=1}^2 V_i^s B_{il}^{ss} \sum_{j=1}^2 A_{lj}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr} \right) Y_1^r + \sum_{i=1}^2 V_i^s B_{i2}^{ss} \sum_{j=1}^2 A_{2j}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr} \right) Y_1^r & 0 \\ \sum_{i=1}^2 V_i^s B_{il}^{ss} \sum_{j=1}^2 A_{lj}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr} \right) Y_2^r + \sum_{i=1}^2 V_i^s B_{i2}^{ss} \sum_{j=1}^2 A_{2j}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr} \right) Y_2^r & 0 \\ \sum_{i=1}^2 V_i^r B_{il}^{rs} \sum_{j=1}^2 A_{lj}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr} \right) Y_1^r + \sum_{i=1}^2 V_i^r B_{i2}^{rs} \sum_{j=1}^2 A_{2j}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr} \right) Y_1^r & 0 \\ \sum_{i=1}^2 V_i^r B_{il}^{rs} \sum_{j=1}^2 A_{lj}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr} \right) Y_2^r + \sum_{i=1}^2 V_i^r B_{i2}^{rs} \sum_{j=1}^2 A_{2j}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr} \right) Y_2^r & 0 \\ \sum_{i=1}^2 V_i^t B_{il}^{ts} \sum_{j=1}^2 A_{lj}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr} \right) Y_1^r + \sum_{i=1}^2 V_i^t B_{i2}^{ts} \sum_{j=1}^2 A_{2j}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr} \right) Y_1^r & 0 \\ \sum_{i=1}^2 V_i^t B_{il}^{ts} \sum_{j=1}^2 A_{lj}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr} \right) Y_2^r + \sum_{i=1}^2 V_i^t B_{i2}^{ts} \sum_{j=1}^2 A_{2j}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr} \right) Y_2^r & 0 \end{array} \right)$$

此矩阵是将上一页的矩阵的第一列与第二列相加、第三列与第四列相加、第五列与第六列相加后得到的

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$\text{rowSums}(\overline{(\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[3:4])^T * \text{rowSums}(\text{Y})[3:4,])[m:n])}[1] = \left(\begin{array}{l} \sum_{i=1}^2 V_i^s B_{i1}^{ss} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r + \sum_{i=1}^2 V_i^s B_{i2}^{ss} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r \\ \sum_{i=1}^2 V_i^s B_{i1}^{ss} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) Y_2^r + \sum_{i=1}^2 V_i^s B_{i2}^{ss} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) Y_2^r \\ \sum_{i=1}^2 V_i^r B_{i1}^{rs} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r + \sum_{i=1}^2 V_i^r B_{i2}^{rs} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r \\ \sum_{i=1}^2 V_i^r B_{i1}^{rs} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) Y_2^r + \sum_{i=1}^2 V_i^r B_{i2}^{rs} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) Y_2^r \\ \sum_{i=1}^2 V_i^t B_{i1}^{ts} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r + \sum_{i=1}^2 V_i^t B_{i2}^{ts} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r \\ \sum_{i=1}^2 V_i^t B_{i1}^{ts} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) Y_2^r + \sum_{i=1}^2 V_i^t B_{i2}^{ts} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) Y_2^r \end{array} \right)$$

$$\text{dim}(\text{rowSums}(\overline{(\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[3:4])^T * \text{rowSums}(\text{Y})[3:4,])[m:n])}[1]) = c(N, G)$$

$$= \left(\begin{array}{l} \sum_{i=1}^2 V_i^s B_{i1}^{ss} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r + \sum_{i=1}^2 V_i^s B_{i2}^{ss} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r \\ \sum_{i=1}^2 V_i^r B_{i1}^{rs} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r + \sum_{i=1}^2 V_i^r B_{i2}^{rs} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r \\ \sum_{i=1}^2 V_i^t B_{i1}^{ts} \sum_{j=1}^2 A_{1j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r + \sum_{i=1}^2 V_i^t B_{i2}^{ts} \sum_{j=1}^2 A_{2j}^{sr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r \end{array} \right)^T$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$\text{rowSums}(\text{dim}(\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n, :]) * (\text{AmLL}[3:4, :])^T * \text{rowSums}(\text{Y})[3:4, :])[m:n, :])[1]) = c(N, G))$$

$$= \left(\left[\sum_{k=s, i=1}^{t, 2} V_i^k B_{il}^{ks} \sum_{j=1}^2 A_{lj}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr} \right) + \sum_{k=s, i=1}^{t, 2} V_i^k B_{i2}^{ks} \sum_{j=1}^2 A_{2j}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr} \right) \right] Y_1^r \quad \left[\sum_{k=s, i=1}^{t, 2} V_i^k B_{il}^{ks} \sum_{j=1}^2 A_{lj}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr} \right) + \sum_{k=s, i=1}^{t, 2} V_i^k B_{i2}^{ks} \sum_{j=1}^2 A_{2j}^{sr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr} \right) \right] Y_2^r \right)^T$$

$$= \left(\left[\sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{ks} A_{jl}^{sr} \right) \sum_{m=1}^2 L_{lm}^{rr} L_{m1}^{rr} + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{ks} A_{j2}^{sr} \right) \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} \right] Y_1^r \quad \left[\sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{ks} A_{jl}^{sr} \right) \sum_{m=1}^2 L_{lm}^{rr} L_{m2}^{rr} + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{ks} A_{j2}^{sr} \right) \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} \right] Y_2^r \right)^T$$

含义说明:

r国从**s**国进口中间品并用于**r**国部门1参与GVC生产活动中包含的**s**国的增加值以及其他第三国的增加值

此矩阵的出口国为**s**国，直接进口国为**r**国，并且**r**国亦是目的地

将**r**国从**s**国进口中间品的结果放入空矩阵

VBAmLLY_back^r =

$$\left(\begin{array}{cc} \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{ks} A_{jl}^{sr} \right) \sum_{m=1}^2 L_{lm}^{rr} L_{m1}^{rr} Y_1^r & \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{ks} A_{j2}^{sr} \right) \sum_{m=1}^2 L_{lm}^{rr} L_{m2}^{rr} Y_1^r \\ + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{ks} A_{j2}^{sr} \right) \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} Y_1^r & + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{ks} A_{j2}^{sr} \right) \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} Y_2^r \end{array} \quad \begin{array}{cc} 0 & 0 \\ \text{export}^{tr} & \text{export}^{tr} \end{array} \right)^T$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$\text{rowSums}(\overline{(\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[3:4])^T * \text{rowSums}(Y)[3:4,])[m:n])[3]} = \left(\begin{array}{l} \sum_{i=1}^2 V_i^s B_{i1}^{st} \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r + \sum_{i=1}^2 V_i^s B_{i2}^{st} \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r \\ \sum_{i=1}^2 V_i^s B_{i1}^{st} \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) Y_2^r + \sum_{i=1}^2 V_i^s B_{i2}^{st} \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) Y_2^r \\ \sum_{i=1}^2 V_i^r B_{i1}^{rt} \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r + \sum_{i=1}^2 V_i^r B_{i2}^{rt} \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r \\ \sum_{i=1}^2 V_i^r B_{i1}^{rt} \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) Y_2^r + \sum_{i=1}^2 V_i^r B_{i2}^{rt} \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) Y_2^r \\ \sum_{i=1}^2 V_i^t B_{i1}^{tt} \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r + \sum_{i=1}^2 V_i^t B_{i2}^{tt} \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r \\ \sum_{i=1}^2 V_i^t B_{i1}^{tt} \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) Y_2^r + \sum_{i=1}^2 V_i^t B_{i2}^{tt} \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr}) Y_2^r \end{array} \right)$$

$$\text{dim}(\text{rowSums}(\overline{(\text{colSums}(\widehat{\text{VB}}[m:n,]) * (\text{AmLL}[3:4])^T * \text{rowSums}(Y)[3:4,])[m:n])[3]}) = c(N, G)$$

$$= \left(\begin{array}{l} \sum_{i=1}^2 V_i^s B_{i1}^{st} \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r + \sum_{i=1}^2 V_i^s B_{i2}^{st} \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r \\ \sum_{i=1}^2 V_i^r B_{i1}^{rt} \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r + \sum_{i=1}^2 V_i^r B_{i2}^{rt} \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r \\ \sum_{i=1}^2 V_i^t B_{i1}^{tt} \sum_{j=1}^2 A_{1j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r + \sum_{i=1}^2 V_i^t B_{i2}^{tt} \sum_{j=1}^2 A_{2j}^{tr} (\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr}) Y_1^r \end{array} \right)^T$$

Xyd_GVC分解至国家_行业.国家——VBAmLLY分解

$$\text{rowSums}(\text{dim}(\text{rowSums}((\text{colSums}(\widehat{\text{VB}}[m:n, :]) * (\text{AmLL}[3:4])^T * \text{rowSums}(\text{Y})[3:4, :])[m:n])[3]) = c(N, G))$$

$$= \left(\left[\sum_{k=s, i=1}^{t, 2} V_i^k B_{il}^{kt} \sum_{j=1}^2 A_{lj}^{tr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{ml}^{rr} \right) + \sum_{k=s, i=1}^{t, 2} V_i^k B_{i2}^{kt} \sum_{j=1}^2 A_{2j}^{tr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m1}^{rr} \right) \right] Y_1^r \quad \left[\sum_{k=s, i=1}^{t, 2} V_i^k B_{il}^{kt} \sum_{j=1}^2 A_{lj}^{tr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr} \right) + \sum_{k=s, i=1}^{t, 2} V_i^k B_{i2}^{kt} \sum_{j=1}^2 A_{2j}^{tr} \left(\sum_{m=1}^2 L_{jm}^{rr} L_{m2}^{rr} \right) \right] Y_2^r \right)^T$$

$$= \left(\left[\sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kt} A_{jl}^{tr} \right) \sum_{m=1}^2 L_{lm}^{rr} L_{ml}^{rr} + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kt} A_{j2}^{tr} \right) \sum_{m=1}^2 L_{2m}^{rr} L_{m1}^{rr} \right] Y_1^r \quad \left[\sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kt} A_{jl}^{tr} \right) \sum_{m=1}^2 L_{lm}^{rr} L_{m2}^{rr} + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kt} A_{j2}^{tr} \right) \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} \right] Y_2^r \right)^T$$

含义说明:

r国从**t**国进口中间品并用于**r**国部门1参与GVC生产活动中包含的**t**国的增加值以及其他第三国的增加值

此矩阵的出口国为**t**国，直接进口国为**r**国，并且**r**国亦是目的地

将**r**国从**t**国进口中间品的结果放入空矩阵

VBAmLLY_back^r =

$$\begin{pmatrix} \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{ks} A_{jl}^{sr} \right) \sum_{m=1}^2 L_{lm}^{rr} L_{ml}^{rr} Y_1^r & \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{ks} A_{jl}^{sr} \right) \sum_{m=1}^2 L_{lm}^{rr} L_{m2}^{rr} Y_1^r & 0 & 0 & \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kt} A_{jl}^{tr} \right) \sum_{m=1}^2 L_{lm}^{rr} L_{ml}^{rr} Y_1^r & \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kt} A_{jl}^{tr} \right) \sum_{m=1}^2 L_{lm}^{rr} L_{m2}^{rr} Y_2^r \\ + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{ks} A_{j2}^{sr} \right) \sum_{m=1}^2 L_{2m}^{rr} L_{ml}^{rr} Y_1^r & + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{ks} A_{j2}^{sr} \right) \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} Y_2^r & & & + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kt} A_{j2}^{tr} \right) \sum_{m=1}^2 L_{2m}^{rr} L_{ml}^{rr} Y_1^r & + \sum_{k=s, i=1}^{t, 2} V_i^k \left(\sum_{j=1}^2 B_{ij}^{kt} A_{j2}^{tr} \right) \sum_{m=1}^2 L_{2m}^{rr} L_{m2}^{rr} Y_2^r \end{pmatrix}^T$$



备注：后续的计算格式与前边一致.

十一：Xyi_GVC(即PLy_GVC分子)分解至国家_行业.世界——对VBBA_mLY分解

#简单GVC引致的国际总产出

```
Xyi_GVC_S<-colSums(diag(VC)%*%L%*%Am%*%L%*%L%*%diag(rowSums(Yd)))
```

#复杂GVC引致的国际总产出

```
z1 <- diag(VC)%*%B%*%B%*%Am
```

```
zm <- z1
```

```
for (s in 1:G){
```

```
  m <- 1+( s-1 )*N; n <- N+( s-1 )*N
```

```
  zm[ m:n,m:n ] <- 0
```

```
}
```

```
zd<-z1-zm
```

```
Xyi_GVC_D<- colSums(zd)%*%L%*%diag( rowSums( Y ) )
```

```
Xyi_GVC_FR<-(colSums(zm))%*%L%*%diag( rowSums( Y ) )
```

```
Xyi_GVC_F<-(colSums(zm))%*%L%*%diag( rowSums( Y ) )-Xyi_GVC_S
```

```
Xyi_GVC_world_2<-Xyi_GVC_D+Xyi_GVC_F+Xyi_GVC_S
```

```
Xyi_GVC_world_2<-t(Xyi_GVC_world_2)
```

根据上述代码可以得到GVC生产活动引致的国际总产出代码为：

```
#GVC生产活动引致的国际总产出_世界
```

```
Xyi_GVC_world<-colSums(diag(VC)%*%B%*%B%*%Am)%*%L%*%diag( rowSums( Y ) )
```

```
Xyi_GVC_world<-t(Xyi_GVC_world)
```

Xyi_GVC分解至国家_行业.世界——复杂+简单GVC引致的国际总产出

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} \quad L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} \quad \hat{V} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \quad Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix} = \begin{pmatrix} \sum_{u=s}^t Y_1^{su} \\ \sum_{u=s}^t Y_2^{su} \\ \sum_{u=s}^t Y_1^{ru} \\ \sum_{u=s}^t Y_2^{ru} \\ \sum_{u=s}^t Y_1^{tu} \\ \sum_{u=s}^t Y_2^{tu} \end{pmatrix} \quad \widehat{\text{rowSums}(Y)} = \begin{pmatrix} Y_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_2^t \end{pmatrix} = \begin{pmatrix} \sum_{u=s}^t Y_1^{su} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sum_{u=s}^t Y_2^{su} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{u=s}^t Y_1^{ru} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sum_{u=s}^t Y_2^{ru} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{u=s}^t Y_1^{tu} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sum_{u=s}^t Y_2^{tu} \end{pmatrix}$$

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$$\widehat{\text{LrowSums}(Y)} = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} \begin{pmatrix} Y_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_2^t \end{pmatrix} = \begin{pmatrix} L_{11}^{ss} Y_1^s & L_{12}^{ss} Y_2^s & 0 & 0 & 0 & 0 \\ L_{21}^{ss} Y_1^s & L_{22}^{ss} Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} Y_1^r & L_{12}^{rr} Y_2^r & 0 & 0 \\ 0 & 0 & L_{21}^{rr} Y_1^r & L_{22}^{rr} Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} Y_1^t & L_{12}^{tt} Y_2^t \\ 0 & 0 & 0 & 0 & L_{21}^{tt} Y_1^t & L_{22}^{tt} Y_2^t \end{pmatrix}$$

BB =

$$\begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix}$$

$$BB = \begin{pmatrix} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ut} \end{pmatrix}$$

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$$\hat{V}^{BB} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \begin{pmatrix} \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ut} \\ \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{us} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ur} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ut} & \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ut} \end{pmatrix}$$

$$\hat{V}^{BB} = \begin{pmatrix} V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{us} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{us} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{ur} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ur} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{ut} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ut} \\ V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{us} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{us} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ur} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ur} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ut} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ut} \\ V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{us} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{us} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{ur} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ur} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{ut} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ut} \\ V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{us} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{us} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ur} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ur} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ut} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ut} \\ V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{us} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{us} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{ur} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ur} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{ut} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ut} \\ V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{us} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{us} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ur} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ur} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ut} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ut} \end{pmatrix}$$

$$\hat{V}BBA_m =$$

$$\hat{V}_{\text{BBA}m} =$$

[illegible]

$$\left(\sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{k \neq s, i=1}^{t, 2} B_{ji}^{uk} A_{il}^{ks} \right) \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{k \neq s, i=1}^{t, 2} B_{ji}^{uk} A_{i2}^{ks} \right) \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{k \neq r, i=1}^{t, 2} B_{ji}^{uk} A_{il}^{kr} \right) \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{k \neq r, i=1}^{t, 2} B_{ji}^{uk} A_{i2}^{kr} \right) \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{k \neq t, i=1}^{r, 2} B_{ji}^{uk} A_{il}^{kt} \right) \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{k \neq t, i=1}^{r, 2} B_{ji}^{uk} A_{i2}^{kt} \right) \right.$$

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$$\text{colSums}(\widehat{\text{VBBAm}})\widehat{\text{LrowSums}}(\text{Y}) =$$

$$\begin{pmatrix} \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq s,i=1}^{t,2} B_{ji}^{uk} A_{il}^{ks} \right) & \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq s,i=1}^{t,2} B_{ji}^{uk} A_{i2}^{ks} \right) & \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq r,i=1}^{t,2} B_{ji}^{uk} A_{il}^{kr} \right) & \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq r,i=1}^{t,2} B_{ji}^{uk} A_{i2}^{kr} \right) & \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq t,i=1}^{r,2} B_{ji}^{uk} A_{il}^{kt} \right) & \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq t,i=1}^{r,2} B_{ji}^{uk} A_{i2}^{kt} \right) \\ L_{11}^{ss} Y_1^s & L_{12}^{ss} Y_2^s & 0 & 0 & 0 & 0 \\ L_{21}^{ss} Y_1^s & L_{22}^{ss} Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} Y_1^r & L_{12}^{rr} Y_2^r & 0 & 0 \\ 0 & 0 & L_{21}^{rr} Y_1^r & L_{22}^{rr} Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} Y_1^t & L_{12}^{tt} Y_2^t \\ 0 & 0 & 0 & 0 & L_{21}^{tt} Y_1^t & L_{22}^{tt} Y_2^t \end{pmatrix}$$

$$\text{colSums}(\widehat{\text{VBBAm}})\widehat{\text{LrowSums}}(\text{Y}) =$$

$$\begin{pmatrix} \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq s,i=1}^{t,2} B_{ji}^{uk} A_{il}^{ks} \right) L_{11}^{ss} Y_1^s + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq s,i=1}^{t,2} B_{ji}^{uk} A_{i2}^{ks} \right) L_{21}^{ss} Y_1^s \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq s,i=1}^{t,2} B_{ji}^{uk} A_{il}^{ks} \right) L_{12}^{ss} Y_2^s + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq s,i=1}^{t,2} B_{ji}^{uk} A_{i2}^{ks} \right) L_{22}^{ss} Y_2^s \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq r,i=1}^{t,2} B_{ji}^{uk} A_{il}^{kr} \right) L_{11}^{rr} Y_1^r + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq r,i=1}^{t,2} B_{ji}^{uk} A_{i2}^{kr} \right) L_{21}^{rr} Y_1^r \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq r,i=1}^{t,2} B_{ji}^{uk} A_{il}^{kr} \right) L_{12}^{rr} Y_2^r + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq r,i=1}^{t,2} B_{ji}^{uk} A_{i2}^{kr} \right) L_{22}^{rr} Y_2^r \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq t,i=1}^{r,2} B_{ji}^{uk} A_{il}^{kt} \right) L_{11}^{tt} Y_1^t + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq t,i=1}^{r,2} B_{ji}^{uk} A_{i2}^{kt} \right) L_{21}^{tt} Y_1^t \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq t,i=1}^{r,2} B_{ji}^{uk} A_{il}^{kt} \right) L_{12}^{tt} Y_2^t + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq t,i=1}^{r,2} B_{ji}^{uk} A_{i2}^{kt} \right) L_{22}^{tt} Y_2^t \end{pmatrix}^T$$

分解思路解析:

t国向s国出口中间品中包含的t国各部门的增加值的生产活动中引致的国际总产出+r国向s国出口中间品中包含的r国各部门的增加值的生产活动中引致的国际总产出

X_{yi}_GVC分解至国家_行业.世界——复杂+简单GVC引致的国际总产出

$$\text{colSums}(\widehat{\text{VBBAm}})\widehat{\text{LrowSums}}(\text{Y})[:,1:2] = \begin{pmatrix} \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq s,i=1}^{t,2} B_{ji}^{uk} A_{il}^{ks} \right) L_{11}^{ss} Y_1^s + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq s,i=1}^{t,2} B_{ji}^{uk} A_{i2}^{ks} \right) L_{21}^{ss} Y_1^s \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq s,i=1}^{t,2} B_{ji}^{uk} A_{i1}^{ks} \right) L_{12}^{ss} Y_2^s + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq s,i=1}^{t,2} B_{ji}^{uk} A_{i2}^{ks} \right) L_{22}^{ss} Y_2^s \end{pmatrix}^T$$



以第一列与第二列为例：

k国各部门对s国出口中间品中包含的世界各部门的增加值被s国部门1、部门2吸收，因此有：

增加值来源国与部门：世界各国各部门

出口国部门：k国各部门

目的地：s国部门1、s国部门2



$$\text{colSums}(\widehat{\text{VBBAm}})\widehat{\text{LrowSums}}(\text{Y}) = \begin{pmatrix} 0 \\ 0 \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i1}^{rs} \right) L_{11}^{ss} Y_1^s + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i2}^{rs} \right) L_{21}^{ss} Y_1^s \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i1}^{rs} \right) L_{12}^{ss} Y_2^s + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i2}^{rs} \right) L_{22}^{ss} Y_2^s \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{i1}^{ts} \right) L_{11}^{ss} Y_1^s + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{i2}^{ts} \right) L_{21}^{ss} Y_1^s \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{i1}^{ts} \right) L_{12}^{ss} Y_2^s + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{i2}^{ts} \right) L_{22}^{ss} Y_2^s \end{pmatrix}^T$$

Xyi_GVC分解至国家_行业.世界——复杂+简单GVC引致的国际总产出

$$\text{colSums}(\widehat{\text{VBBAm}})\widehat{\text{LrowSums}}(\text{Y})[3:4] = \begin{pmatrix} \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq r,i=1}^{t,2} B_{ji}^{uk} A_{il}^{kr} \right) L_{11}^{rr} Y_1^r + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq r,i=1}^{t,2} B_{ji}^{uk} A_{i2}^{kr} \right) L_{21}^{rr} Y_1^r \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq r,i=1}^{t,2} B_{ji}^{uk} A_{i2}^{kr} \right) L_{12}^{rr} Y_2^r + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq r,i=1}^{t,2} B_{ji}^{uk} A_{i1}^{kr} \right) L_{22}^{rr} Y_2^r \end{pmatrix}^T$$



以第三列与第四列为例：

k国各部门对r国出口中间品中包含的世界各部门的增加值被r国部门1、部门2吸收，因此有：

增加值来源国与部门：世界各国各部门

出口国部门：k国各部门

目的地：r国部门1、r国部门2



$$\text{colSums}(\widehat{\text{VBBAm}})\widehat{\text{LrowSums}}(\text{Y}) = \begin{pmatrix} \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{il}^{sr} \right) L_{11}^{rr} Y_1^r + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{i2}^{sr} \right) L_{21}^{rr} Y_1^r \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{i2}^{sr} \right) L_{12}^{rr} Y_2^r + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{i1}^{sr} \right) L_{22}^{rr} Y_2^r \\ 0 \\ 0 \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{il}^{tr} \right) L_{11}^{rr} Y_1^r + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{i2}^{tr} \right) L_{21}^{rr} Y_1^r \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{i2}^{tr} \right) L_{12}^{rr} Y_2^r + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{i1}^{tr} \right) L_{22}^{rr} Y_2^r \end{pmatrix}^T$$

Xyi_GVC分解至国家_行业.世界——复杂+简单GVC引致的国际总产出

$$\text{colSums}(\widehat{\text{VBBAm}})\widehat{\text{LrowSums}}(\text{Y})[5:6] = \begin{pmatrix} \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq s,i=1}^{t,2} B_{ji}^{uk} A_{il}^{kt} \right) L_{11}^{tt} Y_1^t + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq s,i=1}^{t,2} B_{ji}^{uk} A_{i2}^{kt} \right) L_{21}^{tt} Y_1^t \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq s,i=1}^{t,2} B_{ji}^{uk} A_{il}^{kt} \right) L_{12}^{tt} Y_2^t + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{k \neq s,i=1}^{t,2} B_{ji}^{uk} A_{i2}^{kt} \right) L_{22}^{tt} Y_2^t \end{pmatrix}^T$$

以第五列与第六列为例：

k国各部门对**t**国出口中间品中包含的世界各部门的增加值被**t**国部门1、部门2吸收，因此有：

增加值来源国与部门：世界各国各部门

出口国部门：**k**国各部门

目的地：**t**国部门1、**r**国部门2

$$\text{colSums}(\widehat{\text{VBBAm}})\widehat{\text{LrowSums}}(\text{Y}) = \begin{pmatrix} \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{il}^{st} \right) L_{11}^{tt} Y_1^t + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{i2}^{st} \right) L_{21}^{tt} Y_1^t \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{il}^{st} \right) L_{12}^{tt} Y_2^t + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{i2}^{st} \right) L_{22}^{tt} Y_2^t \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{il}^{rt} \right) L_{11}^{tt} Y_1^t + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i2}^{rt} \right) L_{21}^{tt} Y_1^t \\ \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{il}^{rt} \right) L_{12}^{tt} Y_2^t + \sum_{h=s,m=1}^{t,2} V_m^h \sum_{u=s,j=1}^{t,2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i2}^{rt} \right) L_{22}^{tt} Y_2^t \\ 0 \\ 0 \end{pmatrix}^T$$

十二：Xyi_GVC(即PLy_GVC分子)分解至国家_行业.目的地——对VBBA mLY分解

```
VBB<-diag(VC)%*%B%*%B
VBB_3<-array(0,dim=c(GN,GN))
for (ii in 1:G) {
  m=1+N*(ii-1) ; n=N+(ii-1)*N
  VBB_1<-t(as.matrix(colSums(VBB[m:n,])))
  VBB_2<-matrix(VBB_1,nrow=N,ncol=GN,byrow=TRUE)
  VBB_3[m:n,1:GN]<-VBB_2[1:N,]
}
AmL<-Am%*%L
VBBA mLY_2<-array(0,dim=c(GN,GN))
VBBA mLY_back<-array(0,dim=c(GN*G,1))
VBBA mLY_5<-array(0,dim=c(GN,1))
for (i in 1:G) {
  m=1+N*(i-1) ; n=N+(i-1)*N
  mm=1+GN*(i-1) ; nn=GN+(i-1)*GN
  AmL_1<-t(array(AmL[,m:n],dim=c(GN,GN)))
  VBBA mLY<-VBB_3*AmL_1
  YY<-as.matrix(rowSums(Y))
  YY_1<-matrix(YY[m:n,],nrow=GN,ncol=GN,byrow = FALSE)
  VBBA mLY<-VBBA mLY*YY_1
  for (jj in 1:G){
    ss=1+N*(jj-1) ; uu=N+(jj-1)*N
    VBBA mLY_2<-as.matrix(rowSums(VBBA mLY[,ss:uu]))
    VBBA mLY_3<-VBBA mLY_2
    dim(VBBA mLY_3)<-c(N,G)
    VBBA mLY_4<-as.matrix(rowSums(VBBA mLY_3))
    VBBA mLY_5[ss:uu,1]<-VBBA mLY_4[,1]
  }
  VBBA mLY_back[mm:nn,1]<-VBBA mLY_5[1:GN,1]
}
```

Xyi_GVC分解至国家_行业.目的地——复杂+简单GVC引致的国际总产出

$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} \quad L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} \quad \hat{V} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \quad Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix} = \begin{pmatrix} \sum_{u=s}^t Y_1^{su} \\ \sum_{u=s}^t Y_2^{su} \\ \sum_{u=s}^t Y_1^{ru} \\ \sum_{u=s}^t Y_2^{ru} \\ \sum_{u=s}^t Y_1^{tu} \\ \sum_{u=s}^t Y_2^{tu} \end{pmatrix} \quad \widehat{\text{rowSums}(Y)} = \begin{pmatrix} Y_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_2^t \end{pmatrix} = \begin{pmatrix} \sum_{u=s}^t Y_1^{su} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sum_{u=s}^t Y_2^{su} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{u=s}^t Y_1^{ru} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sum_{u=s}^t Y_2^{ru} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{u=s}^t Y_1^{tu} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sum_{u=s}^t Y_2^{tu} \end{pmatrix}$$

X_{yi}_GVC分解至国家_行业.目的地——复杂+简单GVC引致的国际总产出

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix} \quad \text{rowSums}(Y)[1:2,] = \begin{pmatrix} Y_1^s \\ Y_2^s \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \overline{\text{rowSums}(Y)[1:2,]} = \begin{pmatrix} Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \end{pmatrix}$$

$$\text{rowSums}(Y)[3:4,] = \begin{pmatrix} 0 \\ 0 \\ Y_1^r \\ Y_2^r \\ 0 \\ 0 \end{pmatrix} \quad \overline{\text{rowSums}(Y)[3:4,]} = \begin{pmatrix} Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \end{pmatrix}$$

$$\text{rowSums}(Y)[5:6,] = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ Y_1^t \\ Y_2^t \end{pmatrix} \quad \overline{\text{rowSums}(Y)[5:6,]} = \begin{pmatrix} Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \\ Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \\ Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \end{pmatrix}$$

Xyi_GVC分解至国家_行业.目的地——复杂+简单GVC引致的国际总产出

$$\widehat{VBB} = \begin{pmatrix} V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{us} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{us} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{ur} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ur} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{ut} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ut} \\ V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{us} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{us} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ur} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ur} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ut} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ut} \\ V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{us} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{us} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{ur} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ur} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{ut} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ut} \\ V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{us} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{us} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ur} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ur} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ut} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ut} \\ V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{us} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{us} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{ur} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ur} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{ut} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ut} \\ V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{us} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{us} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ur} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ur} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ut} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ut} \end{pmatrix}$$

$$\text{colSums}(\widehat{VBB}[m:n,]) =$$

$$\begin{pmatrix} V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{us} + V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{us} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{us} + V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{us} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{ur} + V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ur} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ur} + V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ur} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{ut} + V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ut} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ut} + V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ut} \\ V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{us} + V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{us} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{us} + V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{us} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{ur} + V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ur} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ur} + V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ur} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{ut} + V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ut} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ut} + V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ut} \\ V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{us} + V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{us} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{us} + V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{us} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{ur} + V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ur} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ur} + V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ur} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{ut} + V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ut} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ut} + V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ut} \end{pmatrix}$$

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$$\text{colSums}(\widehat{\text{VBB}}[m:n,]) = \begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \end{pmatrix}$$

将VBB矩阵的第一行与第二行
 加总、第三行与第四行加
 总、第五行与第六行加总

$$\overline{\text{colSums}(\widehat{\text{VBB}}[m:n,])} = \begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \end{pmatrix}$$

Xyi_GVC分解至国家_行业.目的地——复杂+简单GVC引致的国际总产出

$$AmL = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix} \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{sr} L_{j1}^{rr} & \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} & \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} \\ 0 & 0 & \sum_{j=1}^2 A_{2j}^{sr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} & \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} & \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & 0 & 0 & \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} & \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{tr} L_{j1}^{rr} & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & 0 & 0 \\ \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{tr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} & 0 & 0 \end{pmatrix}$$

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$$AmL[,1:2] = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} \\ \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} \\ \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} \\ \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \end{pmatrix}$$

$$\overline{AmL[,1:2]} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} \\ \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} \\ \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} \\ \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \end{pmatrix}$$

$$(\overline{AmL[,1:2]})^T = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \end{pmatrix}$$

Xyi_GVC分解至国家_行业.目的地——复杂+简单GVC引致的国际总产出

$$\text{colSums}(\widehat{\text{VBB}}[m:n,]) * (\overline{\text{AmL}}, 1:2)^T =$$

$$\begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \end{pmatrix} * \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{lj}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{lj}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{lj}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{lj}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{lj}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{lj}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{lj}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{lj}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{lj}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{jl}^{ss} & \sum_{j=1}^2 A_{lj}^{ts} L_{jl}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{jl}^{ss} \\ 0 & 0 & \sum_{j=1}^2 A_{lj}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & \sum_{j=1}^2 A_{lj}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} \end{pmatrix}$$

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$$\overline{\text{colSums}(\widehat{\text{VBB}}[m:n,]) * (\overline{\text{AmL}}[,1:2])^T =$$

$$\begin{pmatrix} 0 & 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i1}^{ss} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i1}^{ss} \\ 0 & 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} \\ 0 & 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i1}^{ss} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i1}^{ss} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i1}^{ss} \\ 0 & 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} \\ 0 & 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i1}^{ss} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i1}^{ss} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i1}^{ss} \\ 0 & 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} \end{pmatrix}$$

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$$\text{colSums}(\widehat{\text{VBB}}[m:n,]) * (\text{AmL}[1:2])^T \text{rowSums}(\text{Y})[1:2,] =$$

$$\begin{pmatrix} 0 & 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{il}^{ss} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{il}^{ss} \\ 0 & 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} \\ 0 & 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{il}^{ss} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{il}^{ss} \\ 0 & 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} \\ 0 & 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{il}^{ss} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{il}^{ss} \\ 0 & 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} \end{pmatrix} * \begin{pmatrix} Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \end{pmatrix}$$

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$$\overline{\text{colSums}(\widehat{\text{VBB}}[m:n,]) * (\text{AmL}[1:2])^T \overline{\text{rowSums}(Y)[1:2,]}} = \begin{pmatrix} 0 & 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} Y_1^s & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} Y_1^s & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} Y_1^s & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i1}^{ss} Y_1^s \\ 0 & 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} Y_2^s & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} Y_2^s & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} Y_2^s & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} Y_2^s \\ 0 & 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} Y_1^s & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} Y_1^s & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} Y_1^s & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{il}^{ss} Y_1^s \\ 0 & 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} Y_2^s & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} Y_2^s & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} Y_2^s & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} Y_2^s \\ 0 & 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} Y_1^s & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} Y_1^s & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} Y_1^s & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{il}^{ss} Y_1^s \\ 0 & 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} Y_2^s & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} Y_2^s & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} Y_2^s & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} Y_2^s \end{pmatrix}$$

$$\overline{\text{rowSums}(\widehat{\text{VBB}}[m:n,]) * (\text{AmL}[1:2])^T \overline{\text{rowSums}(Y)[1:2,][,m:n]}} =$$

$$\begin{pmatrix} 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} Y_1^s & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{il}^{ss} Y_1^s \\ 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} Y_2^s & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} Y_2^s \\ 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} Y_1^s & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{il}^{ss} Y_1^s \\ 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} Y_2^s & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} Y_2^s \\ 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} Y_1^s & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{il}^{ss} Y_1^s \\ 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} Y_2^s & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} Y_2^s \end{pmatrix}$$

此矩阵是将上一页的矩阵的第一列与第二列相加、第三列与第四列相加、第五列与第六列相加后得到的

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$$\text{rowSums}(\text{colSums}(\widehat{\text{VBB}}[m:n, :]) * (\text{AmL}[1:2])^T \text{rowSums}(\text{Y})[1:2, :][m:n])[2] =$$

$$\begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} Y_1^s \\ \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} Y_2^s \\ \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} Y_1^s \\ \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} Y_2^s \\ \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} Y_1^s \\ \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} Y_2^s \end{pmatrix}$$

$$\text{dim}(\text{rowSums}(\text{colSums}(\widehat{\text{VBB}}[m:n, :]) * (\text{AmL}[1:2])^T \text{rowSums}(\text{Y})[1:2, :][m:n])[2]) = c(N, G) =$$

$$\begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} Y_1^s & \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} Y_2^s \\ \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} Y_1^s & \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} Y_2^s \\ \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} Y_1^s & \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} Y_2^s \end{pmatrix}^T$$

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$$\begin{aligned} & \text{rowSums}(\text{dim}(\text{rowSums}(\text{colSums}(\widehat{\text{VBB}}[m:n,]) * (\text{AmL}[1:2])^T \text{rowSums}(\text{Y})[1:2,][m:n])[2]) = c(N, G)) \\ &= \left(\sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{il}^{ss} Y_1^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i1}^{ss} Y_1^s \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ur} \sum_{i=1}^2 A_{li}^{rs} L_{i2}^{ss} Y_2^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ur} \sum_{i=1}^2 A_{2i}^{rs} L_{i2}^{ss} Y_2^s \right)^T \\ &= \left(\sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ur} A_{11}^{rs} L_{11}^{ss} Y_1^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ur} A_{12}^{rs} L_{21}^{ss} Y_1^s \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ur} A_{11}^{rs} L_{12}^{ss} Y_2^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ur} A_{12}^{rs} L_{22}^{ss} Y_2^s \right. \\ & \quad \left. + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ur} A_{21}^{rs} L_{11}^{ss} Y_1^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ur} A_{22}^{rs} L_{21}^{ss} Y_1^s \quad + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ur} A_{21}^{rs} L_{12}^{ss} Y_2^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ur} A_{22}^{rs} L_{22}^{ss} Y_2^s \right)^T \\ &= \left(\sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i1}^{rs} \right) L_{11}^{ss} Y_1^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i2}^{rs} \right) L_{21}^{ss} Y_1^s \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i1}^{rs} \right) L_{12}^{ss} Y_2^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i2}^{rs} \right) L_{22}^{ss} Y_2^s \right)^T \end{aligned}$$

含义说明:

s国从r国进口中间品并用于s国部门1参与GVC生产活动中包含的r国的增加值以及其他第三国的增加值引致的国际总产出

此矩阵的出口国为r国，直接进口国为s国，并且s国亦是目的地

将s国从r国进口中间品的结果放入空矩阵

$\text{VBBLAmY_back}^s =$

$$\left(\begin{array}{cc} 0 & 0 \\ \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i1}^{rs} \right) L_{11}^{ss} Y_1^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i1}^{rs} \right) L_{12}^{ss} Y_2^s \\ \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i2}^{rs} \right) L_{21}^{ss} Y_1^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i2}^{rs} \right) L_{22}^{ss} Y_2^s \end{array} \right)^T \begin{array}{cc} \text{export}^{\text{ts}} & \text{export}^{\text{ts}} \end{array}$$

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$$\text{rowSums}(\overline{\text{colSums}(\widehat{\text{VBB}}[m:n,]) * (\text{AmL}[1:2])^T \text{rowSums}(\text{Y})[1:2, \text{II}, m:n]}][3] = \begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} Y_1^s \\ \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} Y_2^s \\ \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{il}^{ss} Y_1^s \\ \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} Y_2^s \\ \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{il}^{ss} Y_1^s \\ \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} Y_2^s \end{pmatrix}$$

$$\text{dim}(\text{rowSums}(\overline{\text{colSums}(\widehat{\text{VBB}}[m:n,]) * (\text{AmL}[1:2])^T \text{rowSums}(\text{Y})[1:2, \text{II}, m:n]}][3]) = c(N, G) =$$

$$\begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{il}^{ss} Y_1^s & \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} Y_2^s \\ \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{il}^{ss} Y_1^s & \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} Y_2^s \\ \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} Y_1^s + \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{il}^{ss} Y_1^s & \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} Y_2^s + \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} Y_2^s \end{pmatrix}^T$$

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$$\begin{aligned} & \text{rowSums}(\text{dim}(\text{rowSums}(\text{colSums}(\widehat{\text{VBB}}[m:n, :]) * (\text{AmL}[1:2])^T \text{rowSums}(\text{Y})[1:2, :][, m:n])[, 3]) = c(N, G)) \\ &= \left(\sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{il}^{ss} Y_1^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i1}^{ss} Y_1^s \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{ts} L_{i2}^{ss} Y_2^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{ts} L_{i2}^{ss} Y_2^s \right)^T \\ &= \left(\begin{aligned} & \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ut} A_{11}^{ts} L_{11}^{ss} Y_1^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ut} A_{12}^{ts} L_{21}^{ss} Y_1^s \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ut} A_{11}^{ts} L_{12}^{ss} Y_2^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ut} A_{12}^{ts} L_{22}^{ss} Y_2^s \\ & + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ut} A_{21}^{ts} L_{11}^{ss} Y_1^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ut} A_{22}^{ts} L_{21}^{ss} Y_1^s \quad + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ut} A_{21}^{ts} L_{12}^{ss} Y_2^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ut} A_{22}^{ts} L_{22}^{ss} Y_2^s \end{aligned} \right)^T \\ &= \left(\sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{il}^{ts} \right) L_{11}^{ss} Y_1^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{i2}^{ts} \right) L_{21}^{ss} Y_1^s \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{il}^{ts} \right) L_{12}^{ss} Y_2^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{i2}^{ts} \right) L_{22}^{ss} Y_2^s \right)^T \end{aligned}$$

含义说明:

s国从t国进口中间品并用于s国部门1参与GVC生产活动中包含的t国的增加值以及其他第三国的增加值引致的国际总产出

此矩阵的出口国为t国，直接进口国为s国，并且s国亦是目的地

将s国从t国进口中间品的结果放入空矩阵

VBBLAmY_back^s =

$$\begin{pmatrix} \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{il}^{rs} \right) L_{11}^{ss} Y_1^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i2}^{rs} \right) L_{21}^{ss} Y_1^s & \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{il}^{rs} \right) L_{12}^{ss} Y_2^s + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ur} A_{i2}^{rs} \right) L_{22}^{ss} Y_2^s \\ 0 & 0 \end{pmatrix}^T$$

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$$B = \begin{pmatrix} B_{11}^{ss} & B_{12}^{ss} & B_{11}^{sr} & B_{12}^{sr} & B_{11}^{st} & B_{12}^{st} \\ B_{21}^{ss} & B_{22}^{ss} & B_{21}^{sr} & B_{22}^{sr} & B_{21}^{st} & B_{22}^{st} \\ B_{11}^{rs} & B_{12}^{rs} & B_{11}^{rr} & B_{12}^{rr} & B_{11}^{rt} & B_{12}^{rt} \\ B_{21}^{rs} & B_{22}^{rs} & B_{21}^{rr} & B_{22}^{rr} & B_{21}^{rt} & B_{22}^{rt} \\ B_{11}^{ts} & B_{12}^{ts} & B_{11}^{tr} & B_{12}^{tr} & B_{11}^{tt} & B_{12}^{tt} \\ B_{21}^{ts} & B_{22}^{ts} & B_{21}^{tr} & B_{22}^{tr} & B_{21}^{tt} & B_{22}^{tt} \end{pmatrix} \quad L = \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} \quad \hat{V} = \begin{pmatrix} V_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & V_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & V_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & V_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & V_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & V_2^t \end{pmatrix} \quad Am = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix}$$

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix} = \begin{pmatrix} \sum_{u=s}^t Y_1^{su} \\ \sum_{u=s}^t Y_2^{su} \\ \sum_{u=s}^t Y_1^{ru} \\ \sum_{u=s}^t Y_2^{ru} \\ \sum_{u=s}^t Y_1^{tu} \\ \sum_{u=s}^t Y_2^{tu} \end{pmatrix} \quad \widehat{\text{rowSums}(Y)} = \begin{pmatrix} Y_1^s & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_2^s & 0 & 0 & 0 & 0 \\ 0 & 0 & Y_1^r & 0 & 0 & 0 \\ 0 & 0 & 0 & Y_2^r & 0 & 0 \\ 0 & 0 & 0 & 0 & Y_1^t & 0 \\ 0 & 0 & 0 & 0 & 0 & Y_2^t \end{pmatrix} = \begin{pmatrix} \sum_{u=s}^t Y_1^{su} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sum_{u=s}^t Y_2^{su} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sum_{u=s}^t Y_1^{ru} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sum_{u=s}^t Y_2^{ru} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sum_{u=s}^t Y_1^{tu} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sum_{u=s}^t Y_2^{tu} \end{pmatrix}$$

X_{yi}_GVC分解至国家_行业.目的地——复杂+简单GVC引致的国际总产出

$$\text{rowSums}(Y) = \begin{pmatrix} Y_1^s \\ Y_2^s \\ Y_1^r \\ Y_2^r \\ Y_1^t \\ Y_2^t \end{pmatrix} \quad \text{rowSums}(Y)[1:2,] = \begin{pmatrix} Y_1^s \\ Y_2^s \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \overline{\text{rowSums}(Y)[1:2,]} = \begin{pmatrix} Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \\ Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s & Y_1^s \\ Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s & Y_2^s \end{pmatrix}$$

$$\text{rowSums}(Y)[3:4,] = \begin{pmatrix} 0 \\ 0 \\ Y_1^r \\ Y_2^r \\ 0 \\ 0 \end{pmatrix} \quad \overline{\text{rowSums}(Y)[3:4,]} = \begin{pmatrix} Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \end{pmatrix}$$

$$\text{rowSums}(Y)[5:6,] = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ Y_1^t \\ Y_2^t \end{pmatrix} \quad \overline{\text{rowSums}(Y)[5:6,]} = \begin{pmatrix} Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \\ Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \\ Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t & Y_1^t \\ Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t & Y_2^t \end{pmatrix}$$

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$$\widehat{VBB} = \begin{pmatrix} V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{us} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{us} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{ur} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ur} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{ut} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ut} \\ V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{us} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{us} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ur} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ur} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ut} & V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ut} \\ V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{us} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{us} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{ur} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ur} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{ut} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ut} \\ V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{us} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{us} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ur} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ur} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ut} & V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ut} \\ V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{us} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{us} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{ur} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ur} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{ut} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ut} \\ V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{us} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{us} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ur} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ur} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ut} & V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ut} \end{pmatrix}$$

$$\text{colSums}(\widehat{VBB}[m:n,]) =$$

$$\begin{pmatrix} V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{us} + V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{us} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{us} + V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{us} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{ur} + V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ur} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ur} + V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ur} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{jl}^{ut} + V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{jl}^{ut} & V_1^s \sum_{u=s,j=1}^{t,2} B_{1j}^{su} B_{j2}^{ut} + V_2^s \sum_{u=s,j=1}^{t,2} B_{2j}^{su} B_{j2}^{ut} \\ V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{us} + V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{us} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{us} + V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{us} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{ur} + V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ur} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ur} + V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ur} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{jl}^{ut} + V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{jl}^{ut} & V_1^r \sum_{u=s,j=1}^{t,2} B_{1j}^{ru} B_{j2}^{ut} + V_2^r \sum_{u=s,j=1}^{t,2} B_{2j}^{ru} B_{j2}^{ut} \\ V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{us} + V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{us} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{us} + V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{us} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{ur} + V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ur} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ur} + V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ur} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{jl}^{ut} + V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{jl}^{ut} & V_1^t \sum_{u=s,j=1}^{t,2} B_{1j}^{tu} B_{j2}^{ut} + V_2^t \sum_{u=s,j=1}^{t,2} B_{2j}^{tu} B_{j2}^{ut} \end{pmatrix}$$

Xyi_GVC分解至国家_行业.目的地——复杂+简单GVC引致的国际总产出

$$\text{colSums}(\widehat{\text{VBB}}[m:n,]) = \begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \end{pmatrix}$$

将VBB矩阵的第一行与第二行
 加总、第三行与第四行加
 总、第五行与第六行加总

$$\overline{\text{colSums}(\widehat{\text{VBB}}[m:n,])} = \begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \end{pmatrix}$$

Xyi_GVC分解至国家_行业.目的地——复杂+简单GVC引致的国际总产出

$$AmL = \begin{pmatrix} 0 & 0 & A_{11}^{sr} & A_{12}^{sr} & A_{11}^{st} & A_{12}^{st} \\ 0 & 0 & A_{21}^{sr} & A_{22}^{sr} & A_{21}^{st} & A_{22}^{st} \\ A_{11}^{rs} & A_{12}^{rs} & 0 & 0 & A_{11}^{rt} & A_{12}^{rt} \\ A_{21}^{rs} & A_{22}^{rs} & 0 & 0 & A_{21}^{rt} & A_{22}^{rt} \\ A_{11}^{ts} & A_{12}^{ts} & A_{11}^{tr} & A_{12}^{tr} & 0 & 0 \\ A_{21}^{ts} & A_{22}^{ts} & A_{21}^{tr} & A_{22}^{tr} & 0 & 0 \end{pmatrix} \begin{pmatrix} L_{11}^{ss} & L_{12}^{ss} & 0 & 0 & 0 & 0 \\ L_{21}^{ss} & L_{22}^{ss} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{11}^{rr} & L_{12}^{rr} & 0 & 0 \\ 0 & 0 & L_{21}^{rr} & L_{22}^{rr} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{11}^{tt} & L_{12}^{tt} \\ 0 & 0 & 0 & 0 & L_{21}^{tt} & L_{22}^{tt} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \sum_{j=1}^2 A_{1j}^{sr} L_{j1}^{rr} & \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{1j}^{st} L_{j1}^{tt} & \sum_{j=1}^2 A_{1j}^{st} L_{j2}^{tt} \\ 0 & 0 & \sum_{j=1}^2 A_{2j}^{sr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{st} L_{j1}^{tt} & \sum_{j=1}^2 A_{2j}^{st} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{1j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{1j}^{rs} L_{j2}^{ss} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{rt} L_{j1}^{tt} & \sum_{j=1}^2 A_{1j}^{rt} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{2j}^{rs} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{rs} L_{j2}^{ss} & 0 & 0 & \sum_{j=1}^2 A_{2j}^{rt} L_{j1}^{tt} & \sum_{j=1}^2 A_{2j}^{rt} L_{j2}^{tt} \\ \sum_{j=1}^2 A_{1j}^{ts} L_{j1}^{ss} & \sum_{j=1}^2 A_{1j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{1j}^{tr} L_{j1}^{rr} & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & 0 & 0 \\ \sum_{j=1}^2 A_{2j}^{ts} L_{j1}^{ss} & \sum_{j=1}^2 A_{2j}^{ts} L_{j2}^{ss} & \sum_{j=1}^2 A_{2j}^{tr} L_{j1}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} & 0 & 0 \end{pmatrix}$$

Xyi_GVC分解至国家_行业.目的地——复杂+简单GVC引致的国际总产出

$$AmL[,3:4] = \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} & \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} \\ \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} \\ 0 & 0 \\ 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} \\ \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \end{pmatrix}$$

$$\overline{AmL[,3:4]} = \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} & \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} & \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} & \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} \\ \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} \\ \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \end{pmatrix}$$

$$(\overline{AmL[,3:4]})^T = \begin{pmatrix} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \end{pmatrix}$$

Xyi_GVC分解至国家_行业.目的地——复杂+简单GVC引致的国际总产出

$$\text{colSums}(\widehat{\text{VBB}}[m:n,]) * (\text{AmL}[3:4])^T =$$

$$\left(\begin{array}{cccccc} \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} & \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} & \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{ur} & \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} & \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} & \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} & \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ur} & \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} & \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} & \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \\ \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} & \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ur} & \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} & \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \end{array} \right) * \left(\begin{array}{cccccc} \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{jl}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ \sum_{j=1}^2 A_{1j}^{sr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{j=1}^2 A_{1j}^{tr} L_{j2}^{rr} & \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \end{array} \right)$$

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$$\text{colSums}(\widehat{\text{VBB}}[m:n,]) * (\widehat{\text{AmL}}[,3:4])^T =$$

$$\begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{jl}^{rr} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{j2}^{rr} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{jl}^{rr} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{j2}^{rr} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{jl}^{rr} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{j2}^{rr} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \end{pmatrix}$$

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$$\text{colSums}(\widehat{\text{VBB}}[m:n,]) * (\text{AmL}[3:4])^T \text{rowSums}(Y)[3:4,] =$$

$$\left(\begin{array}{cc} \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{jl}^{rr} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{j2}^{rr} & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{jl}^{rr} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{j2}^{rr} & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} & 0 & 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{jl}^{rr} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} & 0 & 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{j2}^{rr} & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} \end{array} \right) * \begin{pmatrix} Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \\ Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r & Y_1^r \\ Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r & Y_2^r \end{pmatrix}$$

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$$\overline{\text{colSums}(\widehat{\text{VBB}}[m:n,]) * (\text{AmL}[3:4])^T \text{rowSums}(Y)[3:4,]} = \begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} Y_1^r & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r & 0 & 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{jl}^{rr} Y_1^r & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r \\ \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} Y_2^r & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r & 0 & 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{j2}^{rr} Y_2^r & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} Y_1^r & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r & 0 & 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{jl}^{rr} Y_1^r & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} Y_2^r & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r & 0 & 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{j2}^{rr} Y_2^r & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} Y_1^r & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r & 0 & 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{jl}^{rr} Y_1^r & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} Y_2^r & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r & 0 & 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{j2}^{rr} Y_2^r & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r \end{pmatrix}$$

$$\overline{\text{rowSums}(\text{colSums}(\widehat{\text{VBB}}[m:n,]) * (\text{AmL}[3:4])^T \text{rowSums}(Y)[3:4,])[m:n])} = \begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} Y_1^r + \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r & 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{jl}^{rr} Y_1^r + \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r \\ \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} Y_2^r + \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r & 0 & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{j2}^{rr} Y_2^r + \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} Y_1^r + \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r & 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{jl}^{rr} Y_1^r + \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} Y_2^r + \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r & 0 & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{j2}^{rr} Y_2^r + \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} Y_1^r + \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r & 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{jl}^{rr} Y_1^r + \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{jl}^{rr} Y_1^r \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} Y_2^r + \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r & 0 & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{tr} L_{j2}^{rr} Y_2^r + \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{tr} L_{j2}^{rr} Y_2^r \end{pmatrix}$$

此矩阵是将上一页的矩阵的第一列与第二列相加、第三列与第四列相加、第五列与第六列相加后得到的

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$$\text{rowSums}(\text{colSums}(\widehat{\text{VBB}}[m:n, :]) * (\text{AmL}[3:4])^T \text{rowSums}(\text{Y})[3:4, :][m:n])[1] = \begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} Y_1^r + \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r \\ \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} Y_2^r + \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r \\ \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} Y_1^r + \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r \\ \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} Y_2^r + \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r \\ \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} Y_1^r + \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r \\ \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} Y_2^r + \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r \end{pmatrix}$$

$$\dim(\text{rowSums}(\text{colSums}(\widehat{\text{VBB}}[m:n, :]) * (\text{AmL}[3:4])^T \text{rowSums}(\text{Y})[3:4, :][m:n])[1]) = c(N, G) =$$

$$\begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} Y_1^r + \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r & \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} Y_2^r + \sum_{m=1}^2 V_m^s \sum_{u=s, j=1}^{t,2} B_{mj}^{su} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r \\ \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} Y_1^r + \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r & \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} Y_2^r + \sum_{m=1}^2 V_m^r \sum_{u=s, j=1}^{t,2} B_{mj}^{ru} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r \\ \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{jl}^{rr} Y_1^r + \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{jl}^{rr} Y_1^r & \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{jl}^{us} \sum_{j=1}^2 A_{lj}^{sr} L_{j2}^{rr} Y_2^r + \sum_{m=1}^2 V_m^t \sum_{u=s, j=1}^{t,2} B_{mj}^{tu} B_{j2}^{us} \sum_{j=1}^2 A_{2j}^{sr} L_{j2}^{rr} Y_2^r \end{pmatrix}^T$$

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$$\text{rowSums}(\text{dim}(\text{rowSums}(\text{colSums}(\widehat{\text{VBB}}[m:n,]) * (\text{AmL}[3:4])^T \text{rowSums}(Y)[3:4,][m:n])[1]) = c(N, G))$$

$$= \left(\sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{us} \sum_{i=1}^2 A_{li}^{sr} L_{il}^{rr} Y_1^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{us} \sum_{i=1}^2 A_{2i}^{sr} L_{i1}^{rr} Y_1^r \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{us} \sum_{i=1}^2 A_{li}^{sr} L_{i2}^{rr} Y_2^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{us} \sum_{i=1}^2 A_{2i}^{sr} L_{i2}^{rr} Y_2^r \right)^T$$

$$= \left(\sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{us} A_{11}^{sr} L_{11}^{rr} Y_1^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{us} A_{12}^{sr} L_{21}^{rr} Y_1^r \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{us} A_{11}^{sr} L_{12}^{rr} Y_2^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{us} A_{12}^{sr} L_{22}^{rr} Y_2^r \right)^T$$

$$+ \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{us} A_{21}^{sr} L_{11}^{rr} Y_1^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{us} A_{22}^{sr} L_{21}^{rr} Y_1^r \quad + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{us} A_{21}^{sr} L_{12}^{rr} Y_2^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{us} A_{22}^{sr} L_{22}^{rr} Y_2^r$$

$$= \left(\sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{il}^{sr} \right) L_{11}^{rr} Y_1^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{i2}^{sr} \right) L_{21}^{rr} Y_1^r \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{il}^{sr} \right) L_{12}^{rr} Y_2^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{i2}^{sr} \right) L_{22}^{rr} Y_2^r \right)^T$$

含义说明:

r国从s国进口中间品并用于r国部门1参与GVC生产活动中包含的s国的增加值以及其他第三国的增加值引致的国际总产出

此矩阵的出口国为s国, 直接进口国为r国, 并且r国亦是目的地

将r国从s国进口中间品的结果放入空矩阵

$$\text{VBBLAmY_back}^r =$$

$$\left(\sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{il}^{sr} \right) L_{11}^{rr} Y_1^r \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{il}^{sr} \right) L_{11}^{rr} Y_1^r \right. \\ \left. + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{i2}^{sr} \right) L_{21}^{rr} Y_1^r \quad + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{i2}^{sr} \right) L_{21}^{rr} Y_1^r \right. \\ \left. 0 \quad 0 \quad \text{export}^{tr} \quad \text{export}^{tr} \right)^T$$

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$$\text{rowSums}(\text{colSums}(\widehat{\text{VBB}}[m:n]) * (\text{AmL}[3:4])^T \text{rowSums}(\text{Y})[3:4, \text{J}, m:n])[3] = \begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{\text{tr}} L_{jl}^{\text{rr}} Y_1^r + \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{\text{tr}} L_{jl}^{\text{rr}} Y_1^r \\ \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{\text{tr}} L_{j2}^{\text{rr}} Y_2^r + \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{\text{tr}} L_{j2}^{\text{rr}} Y_2^r \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{\text{tr}} L_{jl}^{\text{rr}} Y_1^r + \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{\text{tr}} L_{jl}^{\text{rr}} Y_1^r \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{\text{tr}} L_{j2}^{\text{rr}} Y_2^r + \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{\text{tr}} L_{j2}^{\text{rr}} Y_2^r \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{\text{tr}} L_{jl}^{\text{rr}} Y_1^r + \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{\text{tr}} L_{jl}^{\text{rr}} Y_1^r \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{\text{tr}} L_{j2}^{\text{rr}} Y_2^r + \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{\text{tr}} L_{j2}^{\text{rr}} Y_2^r \end{pmatrix}$$

$$\text{dim}(\text{rowSums}(\text{colSums}(\widehat{\text{VBB}}[m:n]) * (\text{AmL}[3:4])^T \text{rowSums}(\text{Y})[3:4, \text{J}, m:n])[3]) = c(N, G) =$$

$$\begin{pmatrix} \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{\text{tr}} L_{jl}^{\text{rr}} Y_1^r + \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{\text{tr}} L_{jl}^{\text{rr}} Y_1^r & \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{\text{tr}} L_{j2}^{\text{rr}} Y_2^r + \sum_{m=1}^2 V_m^s \sum_{u=s,j=1}^{t,2} B_{mj}^{su} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{\text{tr}} L_{j2}^{\text{rr}} Y_2^r \\ \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{\text{tr}} L_{jl}^{\text{rr}} Y_1^r + \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{\text{tr}} L_{jl}^{\text{rr}} Y_1^r & \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{\text{tr}} L_{j2}^{\text{rr}} Y_2^r + \sum_{m=1}^2 V_m^r \sum_{u=s,j=1}^{t,2} B_{mj}^{ru} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{\text{tr}} L_{j2}^{\text{rr}} Y_2^r \\ \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{\text{tr}} L_{jl}^{\text{rr}} Y_1^r + \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{\text{tr}} L_{jl}^{\text{rr}} Y_1^r & \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{jl}^{ut} \sum_{j=1}^2 A_{lj}^{\text{tr}} L_{j2}^{\text{rr}} Y_2^r + \sum_{m=1}^2 V_m^t \sum_{u=s,j=1}^{t,2} B_{mj}^{tu} B_{j2}^{ut} \sum_{j=1}^2 A_{2j}^{\text{tr}} L_{j2}^{\text{rr}} Y_2^r \end{pmatrix}^T$$

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$$\begin{aligned} & \text{rowSums}(\text{dim}(\text{rowSums}(\text{colSums}(\widehat{\text{VBB}}[m:n, :]) * (\text{AmL}[3:4])^T \text{rowSums}(\text{Y})[3:4, :][m:n]), [3]) = c(N, G)) \\ &= \left(\sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{tr} L_{il}^{rr} Y_1^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{tr} L_{i1}^{rr} Y_1^r \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ut} \sum_{i=1}^2 A_{li}^{tr} L_{i2}^{rr} Y_2^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ut} \sum_{i=1}^2 A_{2i}^{tr} L_{i2}^{rr} Y_2^r \right)^T \\ &= \left(\sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ut} A_{11}^{tr} L_{11}^{rr} Y_1^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ut} A_{12}^{tr} L_{21}^{rr} Y_1^r \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ut} A_{11}^{tr} L_{12}^{rr} Y_2^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{jl}^{ut} A_{12}^{tr} L_{22}^{rr} Y_2^r \right. \\ & \quad \left. + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ut} A_{21}^{tr} L_{11}^{rr} Y_1^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ut} A_{22}^{tr} L_{21}^{rr} Y_1^r \quad + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ut} A_{21}^{tr} L_{12}^{rr} Y_2^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} B_{j2}^{ut} A_{22}^{tr} L_{22}^{rr} Y_2^r \right)^T \\ &= \left(\sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{il}^{tr} \right) L_{11}^{rr} Y_1^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{i2}^{tr} \right) L_{21}^{rr} Y_1^r \quad \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{il}^{tr} \right) L_{12}^{rr} Y_2^r + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{i2}^{tr} \right) L_{22}^{rr} Y_2^r \right)^T \end{aligned}$$

含义说明:

r国从**t**国进口中间品并用于**r**国部门1参与GVC生产活动中包含的**t**国的增加值以及其他第三国的增加值引致的国际总产出

此矩阵的出口国为**t**国，直接进口国为**r**国，并且**r**国亦是目的地

将**r**国从**t**国进口中间品的结果放入空矩阵

$\text{VBBLAmY_back}^r =$

$$\begin{pmatrix} \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{il}^{sr} \right) L_{11}^{rr} Y_1^r & \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{il}^{sr} \right) L_{11}^{rr} Y_1^r & 0 & 0 & \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{il}^{tr} \right) L_{11}^{rr} Y_1^r & \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{il}^{tr} \right) L_{12}^{rr} Y_2^r \\ + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{i2}^{sr} \right) L_{21}^{rr} Y_1^r & + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{us} A_{i2}^{sr} \right) L_{21}^{rr} Y_1^r & & & + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{i2}^{tr} \right) L_{21}^{rr} Y_1^r & + \sum_{h=s, m=1}^{t, 2} V_m^h \sum_{u=s, j=1}^{t, 2} B_{mj}^{hu} \left(\sum_{i=1}^2 B_{ji}^{ut} A_{i2}^{tr} \right) L_{22}^{rr} Y_2^r \end{pmatrix}^T$$



备注：后续的计算格式与前边一致.